Trigonometrical Ratios of Standard Angles

Ex No: 27.1

Solution 1.

(i) $\sin 60^\circ \sin 30^\circ + \cos 30^\circ \cos 60^\circ.$

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$
$$\sin 30^\circ = \frac{1}{2}$$
$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$
$$\cos 60^\circ = \frac{1}{2}$$
$$\sin 60^\circ \sin 30^\circ + \cos 30^\circ \cos 60^\circ = \frac{\sqrt{3}}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{2}$$
$$= \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$$

(ii) $\sec 30^\circ \operatorname{cosec} 60^\circ + \cos 60^\circ \sin 30^\circ.$

$$\cos 30^\circ = \frac{\sqrt{3}}{2} \Rightarrow \sec 30^\circ = \frac{2}{\sqrt{3}}$$
$$\sin 60^\circ = \frac{\sqrt{3}}{2} \Rightarrow \operatorname{cosec} 60^\circ = \frac{2}{\sqrt{3}}$$
$$\cos 60^\circ = \frac{1}{2}, \sin 30^\circ = \frac{1}{2}$$
$$\sec 30^\circ \operatorname{cosec} 60^\circ + \cos 60^\circ \sin 30^\circ.$$
$$= \frac{2}{\sqrt{3}} \times \frac{2}{\sqrt{3}} + \frac{1}{2} \times \frac{1}{2}$$
$$= \frac{4}{3} + \frac{1}{4} = \frac{16+3}{12} = \frac{19}{12}$$

(iii) $\sec 45^\circ \sin 45^\circ - \sin 30^\circ \sec 60^\circ.$

$$\cos 45^\circ = \frac{1}{\sqrt{2}} \Rightarrow \sec 45^\circ = \sqrt{2}$$
$$\sin 45^\circ = \frac{1}{\sqrt{2}}$$
$$\sin 30^\circ = \frac{1}{2}$$
$$\cos 60^\circ = \frac{1}{2} \Rightarrow \sec 60^\circ = 2$$
$$\sec 45^\circ \sin 45^\circ - \sin 30^\circ \sec 60^\circ$$
$$= \sqrt{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times 2 = 1 - 1 = 0$$

(iv) $\sin^2 30° \sin^2 45° + \sin^2 60° \sin^2 90°$

$$\sin 30° = \frac{1}{2}$$

$$\sin 45° = \frac{1}{\sqrt{2}}$$

$$\sin 60° = \frac{\sqrt{3}}{2}$$

$$\sin 90° = 1$$

$$\sin^2 30° \sin^2 45° + \sin^2 60° \sin^2 90°$$

$$= \left(\frac{1}{2}\right)^2 \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 1 = \frac{1}{4} \times \frac{1}{2} + \frac{3}{4}$$

$$= \frac{1}{8} + \frac{3}{4} = \frac{1+6}{8} = \frac{7}{8}$$

(v) $\tan^2 30° + \tan^2 60° + \tan^2 45°$

$$\tan 30° = \frac{1}{\sqrt{3}}$$

$$\tan 60° = \sqrt{3}$$

$$\tan 45° = 1$$

$$\tan^2 30° + \tan^2 60° + \tan^2 45°$$

$$= \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\sqrt{3}\right)^2 + 1$$

$$= \frac{1}{3} + 3 + 1 = \frac{1+9+3}{3} = \frac{13}{3}$$

(vi) $\sin^2 30° \cos^2 45° + 4 \tan^2 30° + \sin^2 90° + \cos^2 0°$

$$\sin 30° = \frac{1}{2}$$

$$\cos 45° = \frac{1}{\sqrt{2}}$$

$$\tan 30° = \frac{1}{\sqrt{3}}$$

$$\sin 90° = 1$$

$$\cos 0° = 1$$

$$\sin^2 30° \cos^2 45° + 4 \tan^2 30° + \sin^2 90° + \cos^2 0°$$

$$= \left(\frac{1}{2}\right)^2 \left(\frac{1}{\sqrt{2}}\right)^2 + 4\left(\frac{1}{\sqrt{3}}\right)^2 + 1 + 1$$

$$= \frac{1}{4} \times \frac{1}{2} + \frac{4}{3} + 2 = \frac{1}{8} + \frac{4}{3} + 2$$

$$= \frac{3+32+48}{24} = \frac{83}{24}$$

(vii) $\operatorname{cosec}^2 45° \sec^2 30° - \sin^2 30° - 4\cot^2 45° + \sec^2 60°$

$$\sin 45° = \frac{1}{\sqrt{2}}$$
$$\operatorname{cosec} 45° = \frac{\sqrt{2}}{1}$$
$$\sin 30° = \frac{1}{2} = \cos 60°$$
$$\sec 60° = 2$$
$$\cos 30° = \frac{\sqrt{3}}{2}$$
$$\sec 30° = \frac{2}{\sqrt{3}}$$
$$\tan 45° = 1$$
$$\cot 45° = 1$$
$$\operatorname{cosec}^2 45° \sec^2 30° - \sin^2 30° - 4\cot^2 45° + \sec^2 60°$$
$$= \left(\frac{\sqrt{2}}{1}\right)^2 \left(\frac{2}{\sqrt{3}}\right)^2 - \left(\frac{1}{2}\right)^2 - 4(1)^2 + (2)^2$$
$$= 2 \times \frac{4}{3} - \frac{1}{4} - 4 + 4 = \frac{8}{3} - \frac{1}{4} = \frac{32-3}{12} = \frac{29}{12}$$

(viii) $\operatorname{cosec}^3 30° \cos 60° \tan^3 45° \sin^2 90° \sec^2 45° \cot 30°$.

$$\sin 30° = \frac{1}{2}$$
$$\operatorname{cosec} 30° = 2$$
$$\cos 60° = \frac{1}{2}$$
$$\sec 60° = 2$$
$$\cos 45° = \frac{1}{\sqrt{2}}$$
$$\sec 45° = \sqrt{2}$$
$$\tan 45° = 1$$
$$\sin 90° = 1$$
$$\tan 30° = \frac{1}{\sqrt{3}} \Rightarrow \cot 30° = \sqrt{3}$$
$$\operatorname{cosec}^3 30° \cos 60° \tan^3 45° \sin^2 90° \sec^2 45° \cot 30°$$
$$= (2)^3 \left(\frac{1}{2}\right)(1)^3 (1)^2 \left(\sqrt{2}\right)^2 \left(\sqrt{3}\right)$$

(ix) $\left(\sin 90^\circ + \sin 45^\circ + \sin 30^\circ\right)\left(\sin 90^\circ - \cos 45^\circ + \cos 60^\circ\right)$.

$$\sin 30^\circ = \frac{1}{2}$$
$$\sin 45^\circ = \frac{1}{\sqrt{2}}$$
$$\sin 90^\circ = 1$$
$$\cos 45^\circ = \frac{1}{\sqrt{2}}$$
$$\cos 60^\circ = \frac{1}{2}$$
$$\left(\sin 90^\circ + \sin 45^\circ + \sin 30^\circ\right)\left(\sin 90^\circ - \cos 45^\circ + \cos 60^\circ\right)$$
$$= \left(1 + \frac{1}{\sqrt{2}} + \frac{1}{2}\right)\left(1 - \frac{1}{\sqrt{2}} + \frac{1}{2}\right)$$
$$= \left(\frac{3}{2} + \frac{1}{\sqrt{2}}\right)\left(\frac{3}{2} - \frac{1}{\sqrt{2}}\right) = \left(\frac{3}{2}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2$$
$$= \frac{9}{4} - \frac{1}{2} = \frac{9-2}{4} = \frac{7}{4}$$

(x) $4\left(\sin^4 30^\circ + \cos^4 60^\circ\right) - 3\left(\cos^2 45^\circ - \sin^2 90^\circ\right)$.

$$\sin 30^\circ = \frac{1}{2}$$
$$\sin 90^\circ = 1$$
$$\cos 45^\circ = \frac{1}{\sqrt{2}}$$
$$\cos 60^\circ = \frac{1}{2}$$
$$4\left(\sin^4 30^\circ + \cos^4 60^\circ\right) - 3\left(\cos^2 45^\circ - \sin^2 90^\circ\right)$$
$$= 4\left(\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4\right) - 3\left(\left(\frac{1}{\sqrt{2}}\right)^2 - (1)^2\right)$$
$$= 4\left(\frac{1}{16} + \frac{1}{16}\right) - 3\left(\frac{1}{2} - 1\right) = 4 \times \frac{2}{16} + 3 \times \frac{1}{2} = \frac{1}{2} + \frac{3}{2} = \frac{4}{2} = 2$$

Solution 2.

(i) $\frac{\sin 30^\circ - \sin 90^\circ + 2\cos 0^\circ}{\tan 30^\circ \tan 60^\circ}$

$$\frac{\sin 30^\circ - \sin 90^\circ + 2\cos 0^\circ}{\tan 30^\circ \tan 60^\circ}$$

$$= \frac{\frac{1}{2} - 1 + 2 \times 1}{\frac{1}{\sqrt{3}} \times \sqrt{3}}$$

$$= \frac{\frac{1}{2} - 1 + 2}{1}$$

$$= \frac{1}{2} - 1 + 2$$

$$= \frac{1}{2} + 1$$

$$= \frac{3}{2}$$

(ii) $\frac{\sin 30^\circ}{\sin 45^\circ} + \frac{\tan 45^\circ}{\sec 60^\circ} - \frac{\sin 60^\circ}{\cot 45^\circ} - \frac{\cos 30^\circ}{\sin 90^\circ}$

$$\frac{\sin 30^\circ}{\sin 45^\circ} + \frac{\tan 45^\circ}{\sec 60^\circ} - \frac{\sin 60^\circ}{\cot 45^\circ} - \frac{\cos 30^\circ}{\sin 90^\circ}$$

$$= \frac{\frac{1}{2}}{\frac{1}{\sqrt{2}}} + \frac{1}{2} - \frac{\frac{\sqrt{3}}{2}}{1} - \frac{\frac{\sqrt{3}}{2}}{1}$$

$$= \frac{\sqrt{2}}{2} + \frac{1}{2} - \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{2} + 1 - 2\sqrt{3}}{2}$$

(iii) $\frac{\tan 45^\circ}{\operatorname{cosec} 30^\circ} + \frac{\sec 60^\circ}{\cot 45^\circ} - \frac{5\sin 90^\circ}{2\cos 0^\circ}$

$$\frac{\tan 45^\circ}{\operatorname{cosec} 30^\circ} + \frac{\sec 60^\circ}{\cot 45^\circ} - \frac{5\sin 90^\circ}{2\cos 0^\circ}$$

$$= \frac{1}{2} + \frac{2}{1} - \frac{5 \times 1}{2 \times 1}$$

$$= \frac{1}{2} + \frac{2}{1} - \frac{5}{2}$$

$$= \frac{1 + 4 - 5}{2}$$

$$= 0$$

(iv) $\frac{\tan^2 60^\circ + 4\cos^2 45^\circ + 3\sec^2 30^\circ + 5\cos 90^\circ}{\operatorname{cosec} 30^\circ + \sec 60^\circ - \cot^2 30^\circ}$

$$\frac{\tan^2 60^\circ + 4\cos^2 45^\circ + 3\sec^2 30^\circ + 5\cos 90^\circ}{\operatorname{cosec} 30^\circ + \sec 60^\circ - \cot^2 30^\circ}$$

$$= \frac{\left(\sqrt{3}\right)^2 + 4 \times \left(\frac{1}{\sqrt{2}}\right)^2 + 3 \times \left(\frac{2}{\sqrt{3}}\right)^2 + 5 \times 0}{2 + 2 - \left(\sqrt{3}\right)^2}$$

$$3 + 4 \times \frac{1}{2} + 3 \times \frac{4}{3} + 0$$

(v) $\frac{4}{\cot^2 30^\circ} + \frac{1}{\sin^2 60^\circ} - \cos^2 45^\circ$

$$\frac{4}{\cot^2 30^\circ} + \frac{1}{\sin^2 60^\circ} - \cos^2 45^\circ$$

$$= \frac{4}{\left(\sqrt{3}\right)^2} + \frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2} - \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= \frac{4}{3} + \frac{1}{\frac{3}{4}} - \frac{1}{2}$$

$$= \frac{4}{3} + \frac{4}{3} - \frac{1}{2}$$

$$= \frac{}{2+2-3}$$

$$= \frac{3+2+4}{4-3}$$

$$= 9$$

$$= \frac{8+8-3}{6}$$

$$= \frac{13}{6}$$

Solution 3a.

$$\text{L.H.S.} = \sin 60^\circ . \cos 30^\circ - \cos 60^\circ . \sin 30^\circ$$

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{3}{4} - \frac{1}{4}$$

$$= \frac{2}{4}$$

$$= \frac{1}{2}$$

$$= \text{R.H.S.}$$

Solution 3b.

$$\text{L.H.S.} = \cos 60^\circ . \cos 30^\circ - \sin 60^\circ . \sin 30^\circ$$

$$= \frac{1}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \times \frac{1}{2}$$

$$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$$

$$= 0$$

$$= \text{R.H.S.}$$

Solution 3c.

$$\text{L.H.S.} = \sec^2 45^\circ - \tan^2 45^\circ$$

$$= \left(\sqrt{2}\right)^2 - (1)^2$$

$$= 2 - 1$$

$$= 1$$

$$= \text{R.H.S.}$$

Solution 3d.

$$\text{L.H.S.} = \left(\frac{\cot 30^\circ + 1}{\cot 30^\circ - 1}\right)^2$$

$$= \left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)^2$$

$$= \left(\frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}\right)^2$$

$$= \frac{\left(\sqrt{3}\right)^2 + (1)^2 + 2\sqrt{3}}{\left(\sqrt{3}\right)^2 + (1)^2 - 2\sqrt{3}}$$

$$= \frac{3+1+2\sqrt{3}}{3+1-2\sqrt{3}}$$

$$= \frac{4+2\sqrt{3}}{4-2\sqrt{3}}$$

$$= \frac{2\left(2+\sqrt{3}\right)}{2\left(2-\sqrt{3}\right)}$$

$$= \frac{2+\sqrt{3}}{2-\sqrt{3}}$$

$$= \frac{\frac{2}{\sqrt{3}}+1}{\frac{2}{\sqrt{3}}-1}$$

$$= \frac{\sec 30^\circ + 1}{\sec 30^\circ - 1}$$

$$= \text{R.H.S.}$$

Solution 4a.

$$2\cos A = 1$$

$$\Rightarrow \cos A = \frac{1}{2}$$

$$\Rightarrow \cos A = \cos 60^\circ$$

$$\Rightarrow A = 60^\circ$$

Solution 4b.

$$2\sin 2A = 1$$
$$\Rightarrow \sin 2A = \frac{1}{2}$$
$$\Rightarrow \sin 2A = \sin 30^\circ$$
$$\Rightarrow 2A = 30^\circ$$
$$\Rightarrow A = 15^\circ$$

Solution 4c.

$$\operatorname{cosec} 3A = \frac{2}{\sqrt{3}}$$
$$\Rightarrow \operatorname{cosec} 3A = \operatorname{cosec} 60^\circ$$
$$\Rightarrow 3A = 60^\circ$$
$$\Rightarrow A = 20^\circ$$

Solution 4d.

$$2\cos 3A = 1$$
$$\Rightarrow \cos 3A = \frac{1}{2}$$
$$\Rightarrow \cos 3A = \cos 60^\circ$$
$$\Rightarrow 3A = 60^\circ$$
$$\Rightarrow A = 20^\circ$$

Solution 4e.

$$\sqrt{3}\cot A = 1$$
$$\Rightarrow \cot A = \frac{1}{\sqrt{3}}$$
$$\Rightarrow \cot A = \cot 60^\circ$$
$$\Rightarrow A = 60^\circ$$

Solution 4f.

$$\cot 3A = 1$$
$$\Rightarrow \cot 3A = \cot 45^\circ$$
$$\Rightarrow 3A = 45^\circ$$
$$\Rightarrow A = 15^\circ$$

Solution 5a.

$$(1 - \operatorname{cosec} A)(2 - \sec A) = 0$$
$$\Rightarrow 1 - \operatorname{cosec} A = 0 \text{ and } 2 - \sec A = 0$$
$$\Rightarrow \operatorname{cosec} A = 1 \text{ and } \sec A = 2$$
$$\Rightarrow \operatorname{cosec} A = \operatorname{cosec} 90° \text{ and } \sec A = \sec 60°$$
$$\Rightarrow A = 90° \text{ and } A = 60°$$

Solution 5b.

$$(2 - \operatorname{cosec} 2A) \cos 3A = 0$$
$$\Rightarrow 2 - \operatorname{cosec} 2A = 0 \text{ and } \cos 3A = 0$$
$$\Rightarrow \operatorname{cosec} 2A = 2 \text{ and } \cos 3A = 0$$
$$\Rightarrow \operatorname{cosec} 2A = \operatorname{cosec} 30° \text{ and } \cos 3A = \cos 90°$$
$$\Rightarrow 2A = 30° \text{ and } 3A = 90°$$
$$\Rightarrow A = 15° \text{ and } A = 30°$$

Solution 6.

$$\sin \alpha + \cos \beta = 1$$
$$\Rightarrow \sin 90° + \cos \beta = 1$$
$$\Rightarrow 1 + \cos \beta = 1$$
$$\Rightarrow \cos \beta = 0$$
$$\Rightarrow \cos \beta = \cos 90°$$
$$\Rightarrow \beta = 90°$$

Solution 7a.

$$\sin \frac{\theta}{3} = 1$$
$$\Rightarrow \sin \frac{\theta}{3} = \sin 90°$$
$$\Rightarrow \frac{\theta}{3} = 90°$$
$$\Rightarrow \theta = 270°$$

Solution 7b.

$$\cot^2(\theta - 5)^\circ = 3$$
$$\Rightarrow \cot(\theta - 5)^\circ = \sqrt{3}$$
$$\Rightarrow \cot(\theta - 5)^\circ = \cot 30^\circ$$
$$\Rightarrow (\theta - 5)^\circ = 30^\circ$$
$$\Rightarrow \theta = 30^\circ + 5^\circ$$
$$\Rightarrow \theta = 35^\circ$$

Solution 7c.

$$\sec\left(\frac{\theta}{2} + 10^\circ\right) = \frac{2}{\sqrt{3}}$$
$$\Rightarrow \sec\left(\frac{\theta}{2} + 10^\circ\right) = \sec 30^\circ$$
$$\Rightarrow \frac{\theta}{2} + 10^\circ = 30^\circ$$
$$\Rightarrow \frac{\theta}{2} = 20^\circ$$
$$\Rightarrow \theta = 40^\circ$$

Solution 8.

(i) $2\sin 3x = \sqrt{3}$

$$2\sin 3x = \sqrt{3}$$
$$\Rightarrow \sin 3x = \frac{\sqrt{3}}{2}$$
$$\Rightarrow \sin 3x = \sin 60^\circ$$
$$\Rightarrow 3x = 60^\circ$$
$$\Rightarrow x = 20^\circ$$

(ii) $2\sin\frac{x}{2} = 1$

$$2\sin\frac{x}{2} = 1$$
$$\Rightarrow \sin\frac{x}{2} = \frac{1}{2}$$
$$\Rightarrow \sin\frac{x}{2} = \sin 30^\circ$$
$$\Rightarrow \frac{x}{2} = 30^\circ$$
$$\Rightarrow x = 60^\circ$$

(iii) $\sqrt{3}\sin x = \cos x$

$$\sqrt{3}\sin x = \cos x$$
$$\Rightarrow \frac{\sin x}{\cos x} = \frac{1}{\sqrt{3}}$$
$$\Rightarrow \tan x = \tan 30^\circ$$
$$\Rightarrow x = 30^\circ$$

(iv) $\tan x = \sin 45^\circ \cos 45^\circ + \sin 30^\circ$

$$\tan x = \sin 45^\circ \cos 45^\circ + \sin 30^\circ$$
$$\Rightarrow \tan x = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{2}$$
$$\Rightarrow \tan x = \frac{1}{2} + \frac{1}{2}$$
$$\Rightarrow \tan x = 1$$
$$\Rightarrow \tan x = \tan 45^\circ$$
$$\Rightarrow x = 45^\circ$$

(v) $\sqrt{3}\tan 2x = \cos 60^\circ + \sin 45^\circ \cos 45^\circ$

$$\sqrt{3}\tan 2x = \cos 60^\circ + \sin 45^\circ \cos 45^\circ$$
$$\Rightarrow \sqrt{3}\tan 2x = \frac{1}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$$
$$\Rightarrow \sqrt{3}\tan 2x = \frac{1}{2} + \frac{1}{2}$$
$$\Rightarrow \sqrt{3}\tan 2x = 1$$
$$\Rightarrow \tan 2x = \frac{1}{\sqrt{3}}$$
$$\Rightarrow \tan 2x = \tan 30^\circ$$
$$\Rightarrow 2x = 30^\circ$$
$$\Rightarrow x = 15^\circ$$

(vi) $\cos 2x = \cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$

$$\cos 2x = \cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$$
$$\Rightarrow \cos 2x = \frac{1}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{2}$$
$$\Rightarrow \cos 2x = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}$$
$$\Rightarrow \cos 2x = \frac{2\sqrt{3}}{4}$$
$$\Rightarrow \cos 2x = \frac{\sqrt{3}}{2}$$
$$\Rightarrow \cos 2x = \cos 30^\circ$$
$$\Rightarrow 2x = 30^\circ$$
$$\Rightarrow x = 15^\circ$$

Solution 9.

$$\sin\theta = \cos\theta$$
$$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{\cos\theta}{\cos\theta}$$
$$\Rightarrow \tan\theta = 1$$
$$\Rightarrow \tan\theta = \tan 45^\circ$$
$$\Rightarrow \theta = 45^\circ$$

Solution 10.

$$\tan\theta = \cot\theta$$
$$\Rightarrow \tan\theta = \frac{1}{\tan\theta}$$
$$\Rightarrow \tan^2\theta = 1$$
$$\Rightarrow \tan\theta = 1$$
$$\Rightarrow \theta = 45^\circ$$

Solution 11.

Given: $\sqrt{2} = 1.414$ and $\sqrt{3} = 1.732$

a. $\tan 60^\circ = \sqrt{3} = 1.732 = 1.73$

b. $\sin 45^\circ \cos 30^\circ = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2\sqrt{2}} = \frac{1.732}{2 \times 1.414} = \frac{1.732}{2.828} = 0.61$

Solution 12a.

Given : $\theta = 30^\circ$

$$\text{L.H.S.} = \tan 2\theta$$
$$= \tan 2 \times 30^\circ$$
$$= \tan 60^\circ$$
$$= \sqrt{3}$$

$$\text{R.H.S.} = \frac{2\tan\theta}{1-\tan^2\theta}$$
$$= \frac{2\tan 30^\circ}{1-\tan^2 30^\circ}$$
$$= \frac{2 \times \frac{1}{\sqrt{3}}}{1-\left(\frac{1}{\sqrt{3}}\right)^2}$$
$$= \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}}$$
$$= \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}}$$
$$= \frac{2}{\sqrt{3}} \times \frac{3}{2}$$
$$= \sqrt{3}$$

$\Rightarrow$ L.H.S. = R.H.S.

$$\Rightarrow \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

Solution 12b.

Given : $\theta = 30^\circ$

$$\text{L.H.S.} = \sin 2\theta$$
$$= \sin 2 \times 30^\circ$$
$$= \sin 60^\circ$$
$$= \frac{\sqrt{3}}{2}$$

$$\text{R.H.S.} = \frac{2\tan\theta}{1+\tan^2\theta}$$
$$= \frac{2\tan 30^\circ}{1+\tan^2 30^\circ}$$
$$= \frac{2 \times \frac{1}{\sqrt{3}}}{1+\left(\frac{1}{\sqrt{3}}\right)^2}$$
$$= \frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}}$$
$$= \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}}$$
$$= \frac{2}{\sqrt{3}} \times \frac{3}{4}$$
$$= \frac{\sqrt{3}}{2}$$

$\Rightarrow$ L.H.S. = R.H.S.

$$\Rightarrow \sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$$

Solution 12c.

Given : $\theta = 30^\circ$

$\cos 2\theta = \cos 2 \times 30^\circ = \cos 60^\circ = \frac{1}{2}$

$$\frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-\tan^2 30^\circ}{1+\tan^2 30^\circ} = \frac{1-\left(\frac{1}{\sqrt{3}}\right)^2}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{1-\frac{1}{3}}{1+\frac{1}{3}} = \frac{\frac{2}{3}}{\frac{4}{3}} = \frac{1}{2}$$

$$\cos^4\theta - \sin^4\theta = \cos^4 30^\circ - \sin^4 30^\circ = \left(\frac{\sqrt{3}}{2}\right)^4 - \left(\frac{1}{2}\right)^4 = \frac{9}{16} - \frac{1}{16} = \frac{8}{16} = \frac{1}{2}$$

$$2\cos^2\theta - 1 = 2\cos^2 30^\circ - 1 = 2\left(\frac{\sqrt{3}}{2}\right)^2 - 1 = 2 \times \frac{3}{4} - 1 = \frac{3}{2} - 1 = \frac{1}{2}$$

$$1 - 2\sin^2\theta = 1 - 2\sin^2 30^\circ = 1 - 2\left(\frac{1}{2}\right)^2 = 1 - 2 \times \frac{1}{4} = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\Rightarrow \cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos^4\theta - \sin^4\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

Solution 12d.

Given : $\theta = 30^\circ$

$$\begin{aligned} \sin 3\theta &= \sin 3 \times 30^\circ \\ &= \sin 90^\circ \\ &= 1 \end{aligned}$$

$$\begin{aligned} 4\sin\theta \sin(60^\circ - \theta)\sin(60^\circ + \theta) &= 4\sin 30^\circ \times \sin(60^\circ - 30^\circ) \times \sin(60^\circ + 30^\circ) \\ &= 4\sin 30^\circ \times \sin 30^\circ \times \sin 90^\circ \\ &= 4 \times \frac{1}{2} \times \frac{1}{2} \times 1 \\ &= 1 \end{aligned}$$

$$\Rightarrow \sin 3\theta = 4\sin\theta \sin(60^\circ - \theta)\sin(60^\circ + \theta)$$

Solution 12e.

Given : $\theta = 30^\circ$

$$1 - \sin 2\theta = 1 - \sin 2 \times 30^\circ$$
$$= 1 - \sin 60^\circ$$
$$= 1 - \frac{\sqrt{3}}{2}$$
$$= \frac{2-\sqrt{3}}{2}$$

$$(\sin\theta - \cos\theta)^2 = \sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta$$
$$= 1 - 2 \times \sin 30^\circ \times \cos 30^\circ$$
$$= 1 - 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2}$$
$$= 1 - \frac{\sqrt{3}}{2}$$
$$= \frac{2-\sqrt{3}}{2}$$

$$\Rightarrow 1 - \sin 2\theta = (\sin\theta - \cos\theta)^2$$

Solution 13a.

$2\theta = 30^\circ$

$\Rightarrow \theta = 15^\circ$

$$\therefore \frac{\sin 3\theta - 2\sin 4\theta}{\cos 3\theta - 2\cos 4\theta} = \frac{\sin 3 \times 15^\circ - 2\sin 4 \times 15^\circ}{\cos 3 \times 15^\circ - 2\cos 4 \times 15^\circ}$$
$$= \frac{\sin 45^\circ - 2\sin 60^\circ}{\cos 45^\circ - 2\cos 60^\circ}$$
$$= \frac{\frac{1}{\sqrt{2}} - 2 \times \frac{\sqrt{3}}{2}}{\frac{1}{\sqrt{2}} - 2 \times \frac{1}{2}}$$
$$= \frac{\frac{1}{\sqrt{2}} - \sqrt{3}}{\frac{1}{\sqrt{2}} - 1}$$
$$= \frac{\frac{1-\sqrt{6}}{\sqrt{2}}}{\frac{1-\sqrt{2}}{\sqrt{2}}}$$
$$= \frac{1-\sqrt{6}}{1-\sqrt{2}}$$
$$= \frac{1-\sqrt{6}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}}$$
$$= \frac{1+\sqrt{2}-\sqrt{6}-\sqrt{12}}{1-2}$$
$$= \frac{1+\sqrt{2}-\sqrt{6}-2\sqrt{3}}{-1}$$
$$= 2\sqrt{3} + \sqrt{6} - \sqrt{2} - 1$$

Solution 13b.

$\theta = 30^\circ$

$$\frac{(1-\cos\theta)(1+\cos\theta)}{(1-\sin\theta)(1+\sin\theta)} = \frac{1-\cos^2\theta}{1-\sin^2\theta}$$

$$= \frac{1-\cos^2 30^\circ}{1-\sin^2 30^\circ}$$

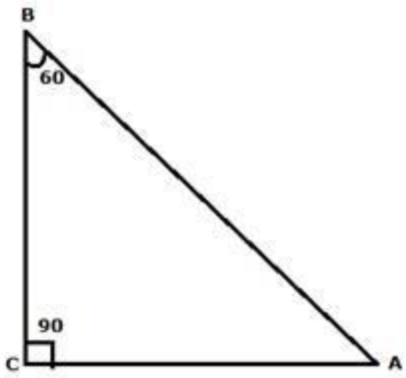
$$= \frac{1-\left(\frac{\sqrt{3}}{2}\right)^2}{1-\left(\frac{1}{2}\right)^2}$$

$$= \frac{1-\frac{3}{4}}{1-\frac{1}{4}}$$

$$= \frac{\frac{1}{4}}{\frac{3}{4}}$$

$$= \frac{1}{3}$$

Solution 14.



$\angle B = 60°$

$\angle C = 90°$ (Since triangle ABC is right angled at C)

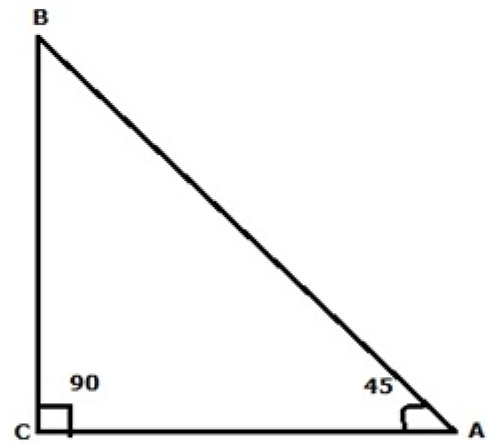
$\angle A + \angle B + \angle C = 180°$
$\angle A + 60° + 90° = 180°$
$\angle A = 180° - 150°$
$\angle A = 30°$

Now,

$\sin 60° = \frac{AC}{AB}$
$\Rightarrow AC = \sin 60° \times AB$
$\Rightarrow AC = \frac{\sqrt{3}}{2} \times 15$
$\Rightarrow AC = \frac{15\sqrt{3}}{2}$ units

Also,
$\cos 60° = \frac{BC}{AB}$
$\Rightarrow BC = \cos 60° \times AB$
$\Rightarrow BC = \frac{1}{2} \times 15$
$\Rightarrow BC = 7.5$ units

Solution 15.



2 $\angle C = 90^\circ, \angle A = 45^\circ$

$\angle A + \angle B + \angle C = 180^\circ$

$45^\circ + \angle B + 90^\circ = 180^\circ$

$\angle B = 180^\circ - 135^\circ$

$\angle B = 45^\circ$

$\sin 45^\circ = \frac{BC}{AB}$

$\Rightarrow AB = \frac{BC}{\sin 45^\circ}$

$\Rightarrow AB = \frac{7}{\frac{1}{\sqrt{2}}}$

$\Rightarrow AB = 7\sqrt{2}\text{units}$

Also,

$\tan 45^\circ = \frac{BC}{AC}$

$\Rightarrow AC = \frac{BC}{\tan 45^\circ}$

$\Rightarrow AC = \frac{7}{1}$

$\Rightarrow AC = 7\text{units}$

Solution 16a.

$A = 30^\circ$ and $B = 60^\circ$

$$\begin{aligned} \text{L.H.S.} &= \sin(A+B) \\ &= \sin(30^\circ + 60^\circ) \\ &= \sin 90^\circ \\ &= 1 \end{aligned}$$

$$\begin{aligned} \text{R.H.S.} &= \sin A \cos B + \cos A \sin B \\ &= \sin 30^\circ \times \cos 60^\circ + \cos 30^\circ \times \sin 60^\circ \\ &= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} \\ &= \frac{1}{4} + \frac{3}{4} \\ &= \frac{4}{4} \\ &= 1 \end{aligned}$$

$\Rightarrow \sin(A+B) = \sin A \cos B + \cos A \sin B$

Solution 16b.

$A = 30^\circ$ and $B = 60^\circ$

$$\begin{aligned} \text{L.H.S.} &= \cos(A+B) \\ &= \cos(30^\circ + 60^\circ) \\ &= \cos 90^\circ \\ &= 0 \end{aligned}$$

$$\begin{aligned} \text{R.H.S.} &= \cos A \cos B - \sin A \sin B \\ &= \cos 30^\circ \times \cos 60^\circ - \sin 30^\circ \times \sin 60^\circ \\ &= \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2} \\ &= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} \\ &= 0 \end{aligned}$$

$\Rightarrow \cos(A+B) = \cos A \cos B - \sin A \sin B$

Solution 16c.

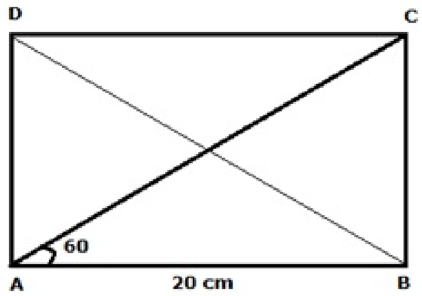
$A = 30^\circ$ and $B = 60^\circ$

$$\text{L.H.S.} = \frac{\sin(A+B)}{\cos A.\cos B}$$
$$= \frac{\sin(30^\circ + 60^\circ)}{\cos 30^\circ \times \cos 60^\circ}$$
$$= \frac{\sin 90^\circ}{\cos 30^\circ \times \cos 60^\circ}$$
$$= \frac{1}{\frac{\sqrt{3}}{2} \times \frac{1}{2}}$$
$$= \frac{4}{\sqrt{3}}$$

$$\text{R.H.S.} = \tan A + \tan B$$
$$= \tan 30^\circ + \tan 60^\circ$$
$$= \frac{1}{\sqrt{3}} + \sqrt{3}$$
$$= \frac{1+3}{\sqrt{3}}$$
$$= \frac{4}{\sqrt{3}}$$

$$\Rightarrow \frac{\sin(A+B)}{\cos A.\cos B} = \tan A + \tan B$$

Solution 16.



In ΔABC,

$$\tan 60^\circ = \frac{BC}{AB}$$
$$\Rightarrow BC = \tan 60^\circ \times AB$$
$$\Rightarrow BC = \sqrt{3} \times 20$$
$$\Rightarrow BC = 20\sqrt{3}cm$$
$$\cos 60^\circ = \frac{AB}{AC}$$
$$\Rightarrow AC = \frac{AB}{\cos 60^\circ}$$
$$\Rightarrow AC = \frac{20}{\frac{1}{2}}$$
$$\Rightarrow AC = 20 \times 2 = 40cm$$

Since diagonals of a rectangle are equal, therefore BD = AC = 40 cm

Solution 16a.

$A = 30^\circ$ and $B = 60^\circ$

$$\text{L.H.S.} = \sin(A+B)$$
$$= \sin(30^\circ + 60^\circ)$$
$$= \sin 90^\circ$$
$$= 1$$

$$\text{R.H.S.} = \sin A \cos B + \cos A \sin B$$
$$= \sin 30^\circ \times \cos 60^\circ + \cos 30^\circ \times \sin 60^\circ$$
$$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$$
$$= \frac{1}{4} + \frac{3}{4}$$
$$= \frac{4}{4}$$
$$= 1$$

$$\Rightarrow \sin(A+B) = \sin A \cos B + \cos A \sin B$$

Solution 16b.

$A = 30^\circ$ and $B = 60^\circ$

L.H.S. $= \cos(A+B)$

$= \cos(30^\circ + 60^\circ)$

$= \cos 90^\circ$

$= 0$

R.H.S. $= \cos A \cos B - \sin A \sin B$

$= \cos 30^\circ \times \cos 60^\circ - \sin 30^\circ \times \sin 60^\circ$

$$= \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$$

$= 0$

$\Rightarrow \cos(A+B) = \cos A \cos B - \sin A \sin B$

Solution 16c.

$A = 30^\circ$ and $B = 60^\circ$

$$\text{L.H.S.} = \frac{\sin(A+B)}{\cos A . \cos B}$$

$$= \frac{\sin(30^\circ + 60^\circ)}{\cos 30^\circ \times \cos 60^\circ}$$

$$= \frac{\sin 90^\circ}{\cos 30^\circ \times \cos 60^\circ}$$

$$= \frac{1}{\frac{\sqrt{3}}{2} \times \frac{1}{2}}$$

$$= \frac{4}{\sqrt{3}}$$

R.H.S. $= \tan A + \tan B$

$= \tan 30^\circ + \tan 60^\circ$

$$= \frac{1}{\sqrt{3}} + \sqrt{3}$$

$$= \frac{1+3}{\sqrt{3}}$$

$$= \frac{4}{\sqrt{3}}$$

$$\Rightarrow \frac{\sin(A+B)}{\cos A . \cos B} = \tan A + \tan B$$

Solution 16d.

$A = 30^\circ$ and $B = 60^\circ$

$$\begin{aligned}
\text{L.H.S.} &= \cos(A+B) \\
&= \cos(30^\circ + 60^\circ) \\
&= \cos 90^\circ \\
&= 0
\end{aligned}$$

$$\begin{aligned}
\text{R.H.S.} &= \cos A \cos B - \sin A \sin B \\
&= \cos 30^\circ \times \cos 60^\circ - \sin 30^\circ \times \sin 60^\circ \\
&= \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2} \\
&= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} \\
&= 0
\end{aligned}$$

$\Rightarrow \cos(A+B) = \cos A \cos B - \sin A \sin B$

Solution 17a.

$A = B = 45^\circ$

$$\begin{aligned}
\text{L.H.S.} &= \sin(A-B) \\
&= \sin(45^\circ - 45^\circ) \\
&= \sin 0^\circ \\
&= 0
\end{aligned}$$

$$\begin{aligned}
\text{R.H.S.} &= \sin A \cos B - \cos A \sin B \\
&= \sin 45^\circ \times \cos 45^\circ - \cos 45^\circ \times \sin 45^\circ \\
&= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \\
&= \frac{1}{2} - \frac{1}{2} \\
&= 0
\end{aligned}$$

$\Rightarrow \sin(A-B) = \sin A \cos B - \cos A \sin B$

Solution 17b.

$A = B = 45°$

L.H.S. $= \cos\ (A - B)$
$= \cos\ (45° - 45°)$
$= \cos\ 0°$
$= 1$

R.H.S. $= \cos\ A\ \cos\ B + \sin\ A\ \sin\ B$
$= \cos\ 45° \times \cos\ 45° + \sin\ 45° \times \sin\ 45°$
$= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$
$= \frac{1}{2} + \frac{1}{2}$
$= 1$

$\Rightarrow \cos\ (A - B) = \cos\ A\ \cos\ B + \sin\ A\ \sin\ B$

Solution 18.

Let $A = 45°$ and $B = 30°$

Then,

$\sin(A - B) = \sin A \cos B - \cos A \sin B$
$\Rightarrow \sin(45° - 30°) = \sin 45° \cos 30° - \cos 45° \sin 30°$
$\Rightarrow \sin 15° = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$
$\Rightarrow \sin 15° = \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$
$\Rightarrow \sin 15° = \frac{(\sqrt{3} - 1)}{2\sqrt{2}}$

$\cos(A - B) = \cos A \cos B + \sin A \sin B$
$\Rightarrow \cos(45° - 30°) = \cos 45° \cos 30° + \sin 45° \sin 30°$
$\Rightarrow \cos 15° = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$
$\Rightarrow \cos 15° = \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$
$\Rightarrow \cos 15° = \frac{(\sqrt{3} + 1)}{2\sqrt{2}}$

Solution 19a.

Since $\theta < 90^\circ$,

Consider $\theta = 45^\circ$

$$\therefore \sin^2\theta + \cos^2\theta = \sin^2 45^\circ + \cos^2 45$$
$$= \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2$$
$$= \frac{1}{2} + \frac{1}{2}$$
$$= 1$$

Solution 19b.

Since $\theta < 90^\circ$,

Consider $\theta = 45^\circ$

$$\therefore \tan^2\theta - \frac{1}{\cos^2\theta} = \tan^2 45^\circ - \frac{1}{\cos^2 45^\circ}$$
$$= (1)^2 - \frac{1}{\left(\frac{1}{\sqrt{2}}\right)^2}$$
$$= 1 - \frac{1}{\frac{1}{2}}$$
$$= 1 - 2$$
$$= -1$$

Solution 20a.

$$\sqrt{3}\sec 2\theta = 2$$
$$\Rightarrow \sec 2\theta = \frac{2}{\sqrt{3}}$$
$$\Rightarrow \sec 2\theta = \sec 30^\circ$$
$$\Rightarrow 2\theta = 30^\circ$$
$$\Rightarrow \theta = 15^\circ$$

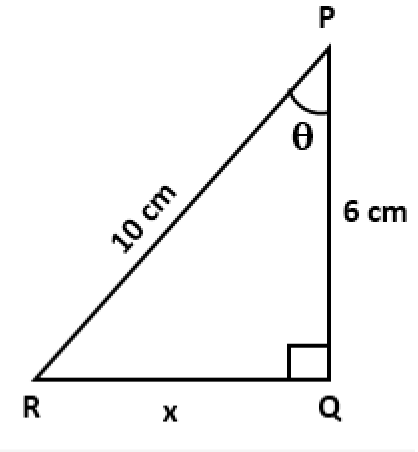
Solution 20b.

$$\sqrt{3}\sec 2\theta = 2$$
$$\Rightarrow \sec 2\theta = \frac{2}{\sqrt{3}}$$
$$\Rightarrow \sec 2\theta = \sec 30^\circ$$
$$\Rightarrow 2\theta = 30^\circ$$
$$\Rightarrow \theta = 15^\circ$$
$$\therefore \cos 3\theta = \cos 3 \times 15^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}}$$

Solution 20c.

$$\sqrt{3}\sec 2\theta = 2$$
$$\Rightarrow \sec 2\theta = \frac{2}{\sqrt{3}}$$
$$\Rightarrow \sec 2\theta = \sec 30^\circ$$
$$\Rightarrow 2\theta = 30^\circ$$
$$\Rightarrow \theta = 15^\circ$$
$$\therefore \cos^2(30^\circ + \theta) + \sin^2(45^\circ - \theta) = \cos^2(30^\circ + 15^\circ) + \sin^2(45^\circ - 15^\circ)$$
$$= \cos^2 45^\circ + \sin^2 30^\circ$$
$$= \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2$$
$$= \frac{1}{2} + \frac{1}{4}$$
$$= \frac{3}{4}$$

Solution 21.



a. $\cos\theta = \dfrac{\text{Base}}{\text{Hypotenuse}}$

$\Rightarrow \cos\theta = \dfrac{PQ}{PR} = \dfrac{6}{10} = \dfrac{3}{5}$

b. $\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \sin^2\theta + \left(\dfrac{3}{5}\right)^2 = 1$

$\Rightarrow \sin^2\theta + \dfrac{9}{25} = 1$

$\Rightarrow \sin^2\theta = 1 - \dfrac{9}{25} = \dfrac{16}{25}$

$\Rightarrow \sin\theta = \dfrac{4}{5}$

$\therefore \sin^2\theta - \cos^2\theta = \dfrac{16}{25} - \dfrac{9}{25} = \dfrac{7}{25}$

c. $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\frac{4}{5}}{\frac{3}{5}} = \dfrac{4}{3}$

But, $\tan\theta = \dfrac{\text{Perpendicular}}{\text{Base}} = \dfrac{RQ}{PQ}$

$\Rightarrow \dfrac{RQ}{PQ} = \dfrac{4}{3}$

$\Rightarrow \dfrac{RQ}{6} = \dfrac{4}{3}$

$\Rightarrow RQ = \dfrac{4 \times 6}{3} = 8\text{ cm}$

Solution 22.

$$\sqrt{\frac{1-\sin^2 60^\circ}{1+\sin^2 60^\circ}} = \sqrt{\frac{1-\left(\frac{\sqrt{3}}{2}\right)^2}{1+\left(\frac{\sqrt{3}}{2}\right)^2}} = \sqrt{\frac{1-\frac{3}{4}}{1+\frac{3}{4}}} = \sqrt{\frac{\frac{1}{4}}{\frac{7}{4}}} = \sqrt{\frac{1}{7}} = \frac{1}{\sqrt{7}}$$

$$3\tan^2\theta - 1 = 0$$
$$\Rightarrow 3\tan^2\theta = 1$$
$$\Rightarrow \tan^2\theta = \frac{1}{3}$$
$$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$$
$$\Rightarrow \tan\theta = \tan 30^\circ$$
$$\Rightarrow \theta = 30^\circ$$

a. $\cos 2\theta = \cos 2 \times 30^\circ = \cos 60^\circ = \frac{1}{2}$

b. $\sin 3\theta = \sin 3 \times 30^\circ = \sin 90^\circ = 1$

Solution 23.

$$\sin(A+B) = 1$$
$$\Rightarrow \sin(A+B) = \sin 90^\circ$$
$$\Rightarrow A+B = 90^\circ \ldots\ldots(i)$$
$$\cos(A-B) = 1$$
$$\Rightarrow \cos(A-B) = \cos 0^\circ$$
$$\Rightarrow A-B = 0^\circ \ldots\ldots\ldots\ldots(ii)$$

adding (i) and (ii)

$$A+B+A-B = 90^\circ + 0^\circ$$
$$2A = 90^\circ$$
$$A = 45^\circ$$

Substituting value of A in (i)

$$A+B = 90^\circ$$
$$45^\circ + B = 90^\circ$$
$$B = 45^\circ$$

Therefore, $A = B = 45^\circ$

Solution 24.

$\tan(A-B)=\frac{1}{\sqrt{3}}$

$\Rightarrow \tan(A-B)=\tan 30^\circ$

$\Rightarrow A-B=30^\circ$......(i)

$\tan(A+B)=\sqrt{3}$

$\Rightarrow \tan(A+B)=\tan 60^\circ$

$\Rightarrow A+B=60^\circ$.........(ii)

Adding (i) and (ii)

$A-B+A+B=30^\circ+60^\circ$

$\Rightarrow 2A=90^\circ$

$\Rightarrow A=45^\circ$

Substituting value of A in (i)

$A-B=30^\circ$

$45^\circ-B=30^\circ$

$B=15^\circ$

Therefore, $A=45^\circ$ and $B=15^\circ$

Solution 25.

$\sin(A-B)=\frac{1}{2}$

$\Rightarrow \sin(A-B)=\sin 30^\circ$

$\Rightarrow A-B=30^\circ$...........(i)

$\cos(A+B)=\frac{1}{2}$

$\Rightarrow \cos(A+B)=\cos 60^\circ$

$\Rightarrow A+B=60^\circ$...........(ii)

Adding (i) and (ii)

$A-B+A+B=30^\circ+60^\circ$

$\Rightarrow 2A=90^\circ$

$\Rightarrow A=45^\circ$

Substituting value of A in (i)

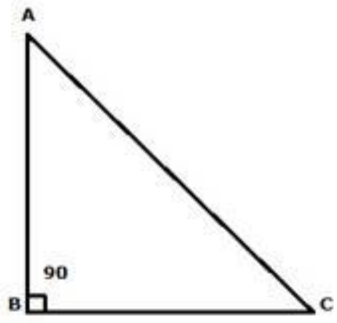
$A-B=30^\circ$

$45^\circ-B=30^\circ$

$B=15^\circ$

Therefore, $A=45^\circ$ and $B=15^\circ$

Solution 26.



Since $\angle B$ is right angled $\Rightarrow \angle B = 90^\circ$

In ΔABC,

$$\angle A + \angle B + \angle C = 180^\circ$$
$$\text{But } \angle A = \angle C$$
$$\Rightarrow \angle A + 90^\circ + \angle A = 180^\circ$$
$$\Rightarrow 2\angle A = 90^\circ$$
$$\Rightarrow \angle A = 45^\circ = \angle C$$

(i)

$$\sin A \cos C + \cos A \sin C$$
$$= \sin 45^\circ \cos 45^\circ + \cos 45^\circ \sin 45^\circ$$
$$= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$$
$$= \frac{1}{2} + \frac{1}{2}$$
$$= 1$$

(ii)

$$\sin A \sin B + \cos A \cos B$$
$$\Rightarrow \sin 45^\circ \sin 90^\circ + \cos 45^\circ \cos 90^\circ$$
$$= \frac{1}{\sqrt{2}} \times 1 + \frac{1}{\sqrt{2}} \times 0$$
$$= \frac{1}{\sqrt{2}}$$

Solution 27.

Since $\tan A = \frac{1}{2}, \tan B = \frac{1}{3}$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\Rightarrow \tan(A+B) = \frac{\frac{1}{2}+\frac{1}{3}}{1-\left(\frac{1}{2}\times\frac{1}{3}\right)}$$

$$\Rightarrow \tan(A+B) = \frac{\frac{5}{6}}{1-\frac{1}{6}}$$

$$\Rightarrow \tan(A+B) = \frac{\frac{5}{6}}{\frac{5}{6}}$$

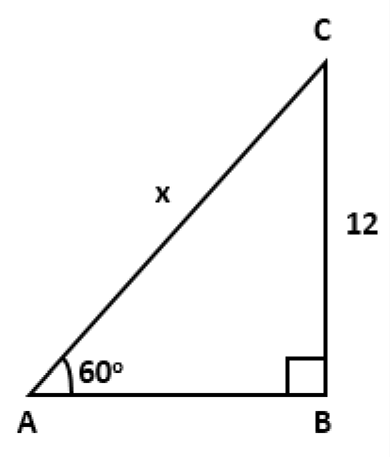
$$\Rightarrow \tan(A+B) = 1$$

$$\Rightarrow \tan(A+B) = \tan 45^\circ$$

$$\Rightarrow A+B = 45^\circ$$

Ex No: 27.2

Solution 1a.



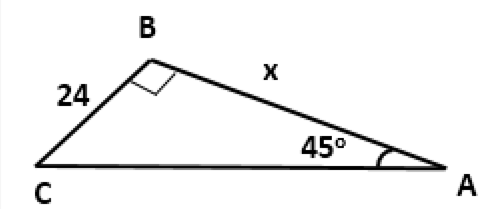
From the figure, we have

$$\sin 60^\circ = \frac{BC}{AC}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{12}{x}$$

$$\Rightarrow x = \frac{2 \times 12}{\sqrt{3}} = \frac{24}{\sqrt{3}} = \frac{8\times 3}{\sqrt{3}} = 8\sqrt{3}$$

Solution 1b.



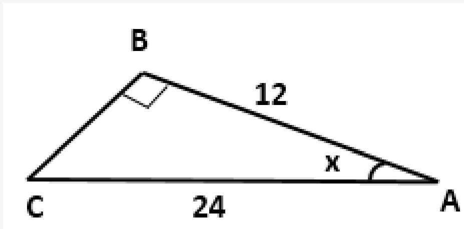
From the figure, we have

$$\tan 45° = \frac{BC}{AB}$$

$$\Rightarrow 1 = \frac{24}{x}$$

$$\Rightarrow x = 24$$

Solution 1c.



From the figure, we have

$$\cos x = \frac{AB}{AC}$$

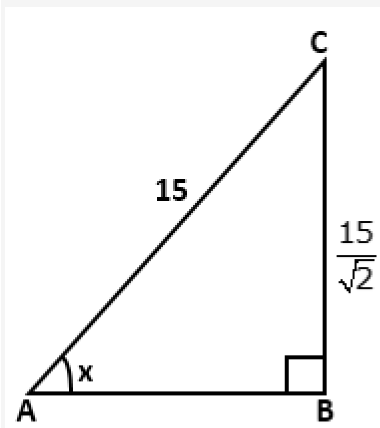
$$\Rightarrow \cos x = \frac{12}{24}$$

$$\Rightarrow \cos x = \frac{1}{2}$$

$$\Rightarrow \cos x = \cos 60°$$

$$\Rightarrow x = 60°$$

Solution 1d.



From the figure, we have

$$\sin x = \frac{BC}{AC}$$

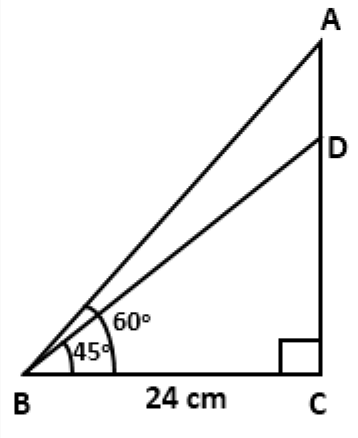
$$\Rightarrow \sin x = \frac{\frac{15}{\sqrt{2}}}{15}$$

$$\Rightarrow \sin x = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sin x = \sin 45°$$

$$\Rightarrow x = 45°$$

Solution 2.



In ΔABC,

$\tan 60° = \frac{AC}{BC}$

$\Rightarrow \sqrt{3} = \frac{AC}{24}$

$\Rightarrow AC = 24\sqrt{3}$ cm

In ΔDBC,

$\tan 45° = \frac{DC}{BC}$

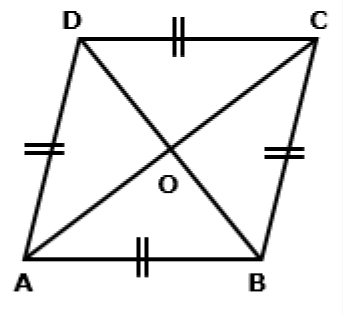
$\Rightarrow 1 = \frac{DC}{24}$

$\Rightarrow DC = 24$ cm

Now, $AC = AD + DC$

$\Rightarrow AD = AC - DC = 24\sqrt{3} - 24 = 24\left(\sqrt{3} - 1\right)$ cm

Solution 3.



The given figure is a rhombus as all sides are equal.
We know that diagonals of a rhombus bisect each other at right angles and also bisect the angle of vertex.
Let the diagonals AC and BD intersect each other at O.

$\Rightarrow OA = OC = \frac{1}{2}AC,\ \ OB = OD = \frac{1}{2}BD,\ \angle AOB = 90^\circ$

Now, $\angle BAD = 60^\circ \Rightarrow \angle OAB = \frac{1}{2}\angle BAD = 30^\circ$

In right-angled ΔAOB,

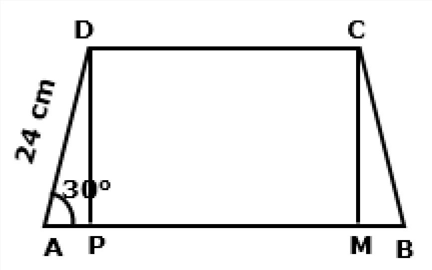
$\sin 30^\circ = \frac{OB}{AB} \Rightarrow \frac{1}{2} = \frac{OB}{24} \Rightarrow OB = 12$ cm

$\cos 30^\circ = \frac{OA}{AB} \Rightarrow \frac{\sqrt{3}}{2} = \frac{OA}{24} \Rightarrow OA = 12\sqrt{3}$ cm

$\therefore$ Length of diagonal $AC = 2 \times OA = 2 \times 12\sqrt{3} = 24\sqrt{3}$ cm
And, length of diagonal $BD = 2 \times OB = 2 \times 12 = 24$ cm

Solution 4.

Construction: Draw DP ⊥ AB and CM ⊥ AB



a. In right ΔADP,

$$\cos 30° = \frac{AP}{AD}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{AP}{24}$$

$$\Rightarrow AP = 12\sqrt{3} \text{ cm}$$

Similarly, from right ΔBCM, we have $MB = 12\sqrt{3}$ cm

Now, in rectangle PMCD, we have CD = PM = 24 cm

∴ Length of $AB = AP + PM + MB = 12\sqrt{3} + 24 + 12\sqrt{3} = 24\left(\sqrt{3} + 1\right)$ cm

b. In right ΔADP,

$$\sin 30° = \frac{PD}{AD}$$

$$\Rightarrow \frac{1}{2} = \frac{PD}{24}$$

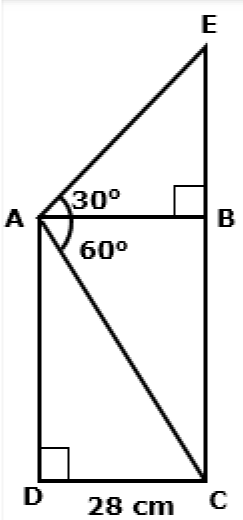
$$\Rightarrow PD = 12 \text{ cm}$$

Similarly, from right ΔBCM, we have $MB = 12\sqrt{3}$ cm

Now, in rectangle PMCD, we have CD = PM = 24 cm

∴ Length of $AB = AP + PM + MB = 12\sqrt{3} + 24 + 12\sqrt{3} = 24\left(\sqrt{3} + 1\right)$ cm

Solution 5.



CD = 28 cm

$\Rightarrow$ AB = 28 cm

In right ΔABE,

$\tan 30° = \frac{BE}{AB}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{BE}{28}$

$\Rightarrow BE = \frac{28}{\sqrt{3}}$

In right ΔABC,

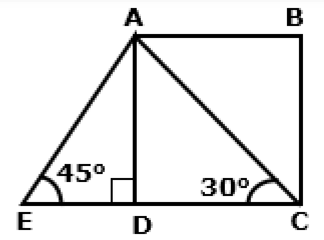
$\tan 60° = \frac{CB}{AB}$

$\Rightarrow \sqrt{3} = \frac{CB}{28}$

$\Rightarrow CB = 28\sqrt{3}$

$\therefore$ Length of EC = CB + BE = $28\sqrt{3} + \frac{28}{\sqrt{3}} = \frac{84+28}{\sqrt{3}} = \frac{112}{\sqrt{3}}$

Solution 6.



a. In right ΔADC,

$$\tan 30° = \frac{AD}{DC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{1.5}{DC}$$

$$\Rightarrow DC = 1.5\sqrt{3}$$

Since $AB \parallel DC$ and $AD \perp EC$, ABCD is a parallelogram and hence opposite sides are equal.

$$\Rightarrow AB = DC = 1.5\sqrt{3} \text{ cm}$$

b. In right ΔADC,

$$\sin 30° = \frac{AD}{AC}$$

$$\Rightarrow \frac{1}{2} = \frac{1.5}{AC}$$

$$\Rightarrow AC = 2 \times 1.5 = 3 \text{ cm}$$

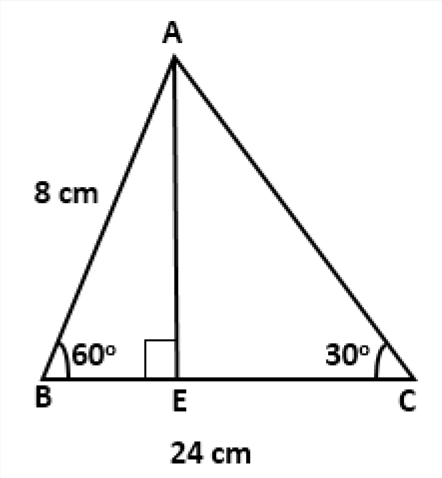
c. In right ΔADE,

$$\sin 45° = \frac{AD}{AE}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{1.5}{AE}$$

$$\Rightarrow AE = 1.5\sqrt{2} \text{ cm}$$

Solution 7.



a. In right ΔAEB,

$$\sin 60^\circ = \frac{AE}{AB}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{AE}{8}$$

$$\Rightarrow AE = 4\sqrt{3} \text{ cm}$$

Now, $BE^2 = AB^2 - AE^2 = 8^2 - \left(4\sqrt{3}\right)^2 = 64 - 48 = 16$

$$\Rightarrow BE = 4 \text{ cm}$$

b. $EC = BC - BE = 24 - 4 = 20 \text{ cm}$

Now, in right ΔAEC,

$$AC^2 = AE^2 + EC^2 = \left(4\sqrt{3}\right)^2 + 20^2 = 48 + 400 = 448$$

$$\Rightarrow AC = 8\sqrt{7} \text{ cm}$$

Solution 8.

a. In right ΔABC,

$$\tan 30° = \frac{AB}{BC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{10}{BC}$$

$$\Rightarrow BC = 10\sqrt{3} \text{ cm}$$

b. In ΔABC, $\angle C = 30°$ and $\angle B = 90°$

$$\Rightarrow \angle A = 60°$$

Now, in ΔABD,

$$\cos 60° = \frac{AD}{AB}$$

$$\Rightarrow \frac{1}{2} = \frac{AD}{10}$$

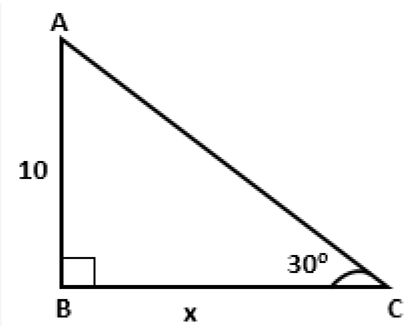
$$\Rightarrow AD = 5 \text{ cm}$$

c. In ΔABC,

$$AC^2 = AB^2 + BC^2 = 10^2 + \left(10\sqrt{3}\right)^2 = 100 + 300 = 400 \text{ cm}$$

$$\Rightarrow AC = 20 \text{ cm}$$

Solution 9a.



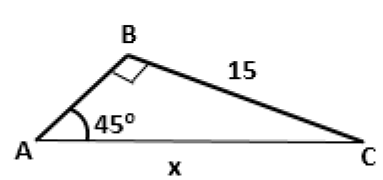
In right ΔABC,

$$\tan 30° = \frac{AB}{BC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{10}{x}$$

$$\Rightarrow x = 10\sqrt{3} \text{ cm}$$

Solution 9b.



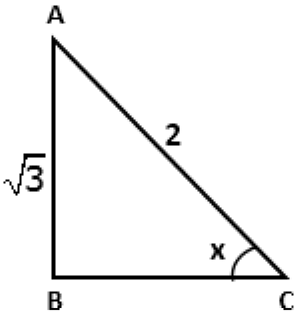
In right ΔABC,

$\sin 45^\circ = \frac{BC}{AC}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{15}{x}$

$\Rightarrow x = 15\sqrt{2}$ cm

Solution 9c.



In right ΔABC,

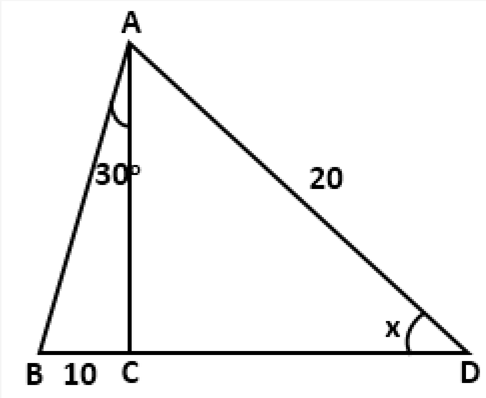
$\sin x = \frac{AB}{AC}$

$\Rightarrow \sin x = \frac{\sqrt{3}}{2}$

$\Rightarrow \sin x = \sin 60^\circ$

$\Rightarrow x = 60^\circ$

Solution 9d.



In right ΔACB,

$$\tan 30^\circ = \frac{BC}{AC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{10}{AC}$$

$$\Rightarrow AC = 10\sqrt{3} \text{ cm}$$

Now, in right ΔACD,

$$\sin x = \frac{AC}{AD}$$

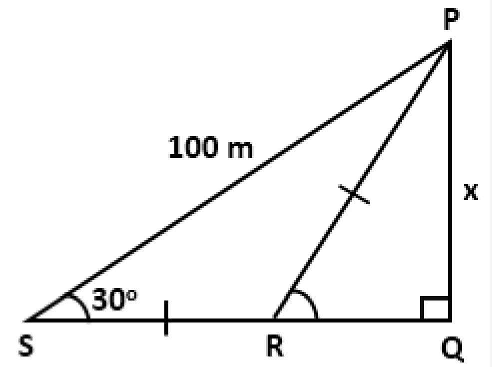
$$\Rightarrow \sin x = \frac{10\sqrt{3}}{20}$$

$$\Rightarrow \sin x = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \sin x = \sin 60^\circ$$

$$\Rightarrow x = 60^\circ$$

Solution 9e.



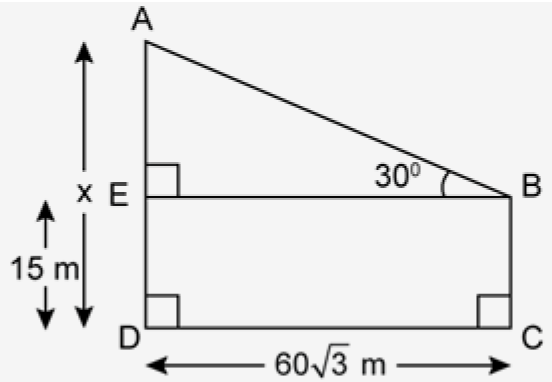
In right ΔPQS,

$\sin 30° = \frac{PQ}{PS}$

$\Rightarrow \frac{1}{2} = \frac{x}{100}$

$\Rightarrow x = 50$ m

Solution 9f.



BEDC is a rectangle.

$\Rightarrow BE = DC = 60\sqrt{3}$ m

In right ΔAEB,

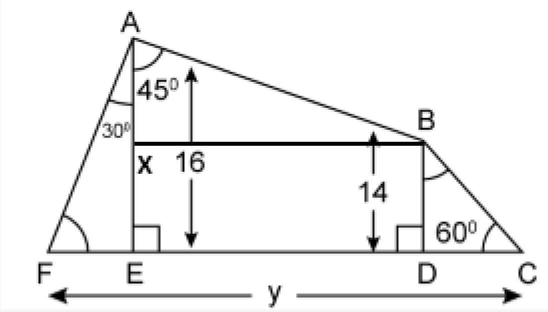
$\tan 30° = \frac{AE}{BE}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AE}{60\sqrt{3}}$

$\Rightarrow AE = 60$ m

Now, $x = AD = AE + ED = 60 + 15 = 75$ m

Solution 10a.



In right ΔAEF,

$\tan 30° = \frac{FE}{AE}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{FE}{16}$

$\Rightarrow FE = \frac{16}{\sqrt{3}}$

In right ΔBDC,

$\tan 60° = \frac{BD}{DC}$

$\Rightarrow \sqrt{3} = \frac{14}{DC}$

$\Rightarrow DC = \frac{14}{\sqrt{3}}$

In right ΔAXB,

$\tan 45° = \frac{BX}{AX}$

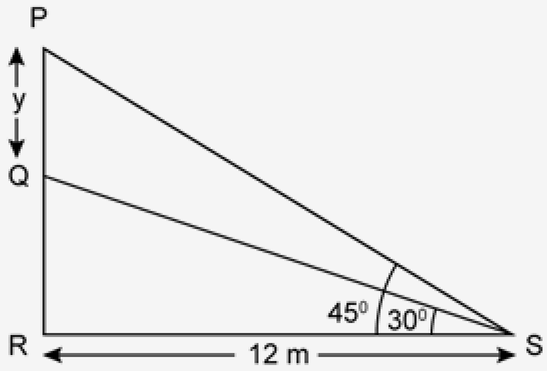
$\Rightarrow 1 = \frac{BX}{2}$

$\Rightarrow BX = 2$

$\Rightarrow ED = BX = 2$

Now, $y = FC = FE + ED + DC = \frac{16}{\sqrt{3}} + \frac{14}{\sqrt{3}} + 2 = \frac{30 + 2\sqrt{3}}{\sqrt{3}} = \frac{30 + 3.464}{1.732} = 19.32$

Solution 10b.



In right ΔPRS,

$\tan 45° = \frac{PR}{RS}$

$\Rightarrow 1 = \frac{PR}{12}$

$\Rightarrow PR = 12$

In right ΔQRS,

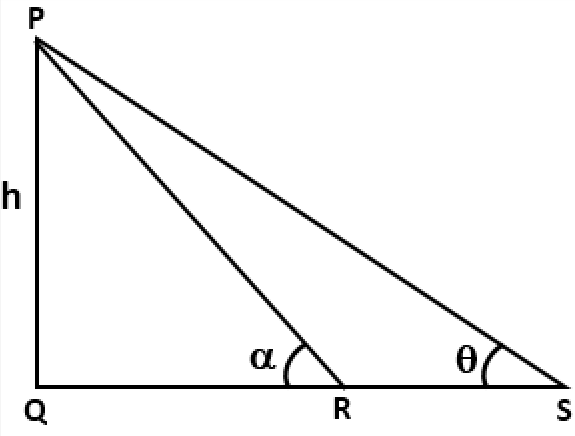
$\tan 30° = \frac{QR}{RS}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{QR}{12}$

$\Rightarrow QR = \frac{12}{\sqrt{3}}$

Now, $y = PQ = PR - QR = 12 - \frac{12}{\sqrt{3}} = 12 - \frac{12}{1.732} = 12 - 6.93 = 5.07$

Solution 11.



$\tan\theta = \frac{PQ}{QS}$

$\Rightarrow \frac{5}{13} = \frac{h}{QS}$

$\Rightarrow 5 \times QS = 13h$

$\Rightarrow 5(QR + RS) = 13h$

$\Rightarrow 5(QR + 12) = 13h$

$\Rightarrow QR + 12 = \frac{13h}{5} \quad \text{....(i)}$

$\tan\alpha = \frac{PQ}{QR}$

$\Rightarrow \frac{3}{5} = \frac{h}{QR}$

$\Rightarrow 3 \times QR = 5h$

$\Rightarrow QR = \frac{5h}{3} \quad \text{....(ii)}$

Substituting (ii) in (i), we have

$\frac{5h}{3} + 12 = \frac{13h}{5}$

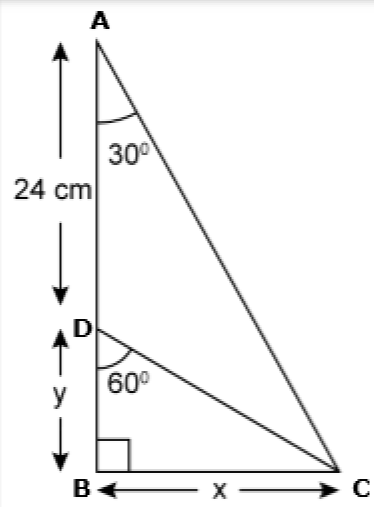
$\Rightarrow \frac{13h}{5} - \frac{5h}{3} = 12$

$\Rightarrow \frac{39h - 25h}{15} = 12$

$\Rightarrow 14h = 180$

$\Rightarrow h = 12.86 \text{ m}$

Solution 12a.



In right ΔABC,

$$\tan 30^\circ = \frac{BC}{AB}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{24 + y} \quad ....(i)$$

In right ΔDBC,

$$\tan 60^\circ = \frac{BC}{DB}$$

$$\Rightarrow \sqrt{3} = \frac{x}{y}$$

$$\Rightarrow x = \sqrt{3}y$$

Substituting the value of x in (i), we get

$$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}y}{24 + y}$$

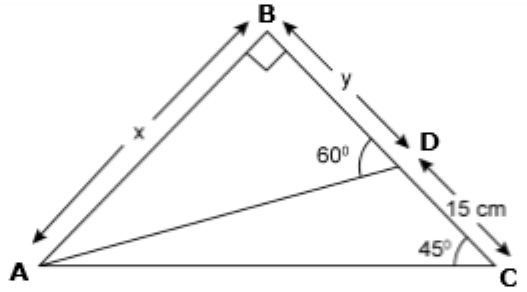
$$\Rightarrow 24 + y = 3y$$

$$\Rightarrow 2y = 24$$

$$\Rightarrow y = 12 \text{ cm}$$

$$\Rightarrow x = \sqrt{3} \times 12 = 12\sqrt{3} \text{ cm}$$

Solution 12b.



In right ΔABC,

$$\tan 45^\circ = \frac{AB}{BC}$$

$$\Rightarrow 1 = \frac{x}{15 + y}$$

$$\Rightarrow x = 15 + y \quad ....(i)$$

In right ΔABD,

$$\tan 60^\circ = \frac{AB}{DB}$$

$$\Rightarrow \sqrt{3} = \frac{x}{y}$$

$$\Rightarrow \sqrt{3} = \frac{15 + y}{y} \quad ....[\text{From (i)}]$$

$$\Rightarrow \sqrt{3}y = 15 + y$$

$$\Rightarrow \sqrt{3}y - y = 15$$

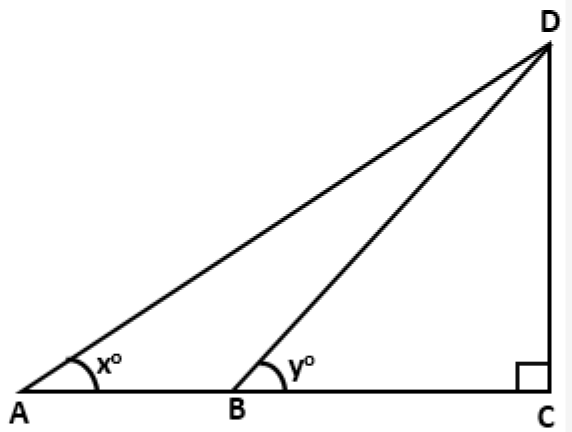
$$\Rightarrow y\left(\sqrt{3} - 1\right) = 15$$

$$\Rightarrow y = \frac{15}{\sqrt{3} - 1}$$

$$\Rightarrow y = \frac{15}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1} = \frac{15\left(\sqrt{3} + 1\right)}{3 - 1} = \frac{15\left(\sqrt{3} + 1\right)}{2} \text{ cm}$$

$$\Rightarrow x = 15 + \frac{15\left(\sqrt{3} + 1\right)}{2} = \frac{30 + 15\left(\sqrt{3} + 1\right)}{2} = \frac{15\left(2 + \sqrt{3} + 1\right)}{2} = \frac{15\left(3 + \sqrt{3}\right)}{2} = \frac{15\sqrt{3}\left(\sqrt{3} + 1\right)}{2}$$

Solution 13.



$$\tan x = \frac{CD}{AC}$$

$$\Rightarrow \frac{5}{12} = \frac{CD}{AC}$$

$$\Rightarrow 5 \times AC = 12 \times CD$$

$$\Rightarrow 5(AB + BC) = 12CD$$

$$\Rightarrow 5(48 + BC) = 12CD$$

$$\Rightarrow 48 + BC = \frac{12CD}{5} \quad ....(i)$$

$$\tan y = \frac{CD}{BC}$$

$$\Rightarrow \frac{3}{4} = \frac{CD}{BC}$$

$$\Rightarrow 3BC = 4CD$$

$$\Rightarrow BC = \frac{4CD}{3} \quad ....(ii)$$

Substituing (ii) in (i), we have

$$48 + \frac{4CD}{3} = \frac{12CD}{5}$$

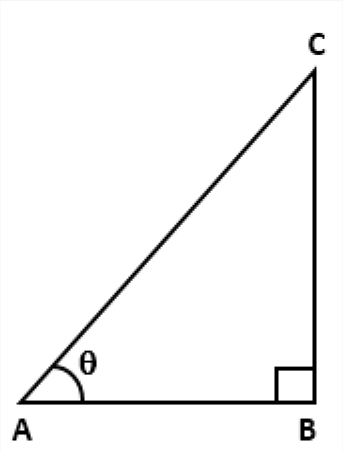
$$\Rightarrow \frac{12CD}{5} - \frac{4CD}{3} = 48$$

$$\Rightarrow \frac{36CD - 20CD}{15} = 48$$

$$\Rightarrow 16CD = 720$$

$$\Rightarrow CD = 45 \text{ m}$$

Solution 17.



a. Given, $AB = \sqrt{3} \times BC$

$\Rightarrow \frac{AB}{BC} = \sqrt{3}$

$\Rightarrow \cot\ \theta = \sqrt{3}$

$\Rightarrow \cot\ \theta = \cot\ 30^\circ$

$\Rightarrow \theta = 30^\circ$

b. Given, $BC = \sqrt{3} \times AB$

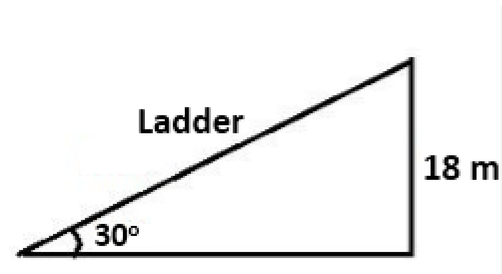
$\Rightarrow \frac{BC}{AB} = \sqrt{3}$

$\Rightarrow \tan\ \theta = \sqrt{3}$

$\Rightarrow \tan\ \theta = \tan\ 60^\circ$

$\Rightarrow \theta = 60^\circ$

Solution 18.



Suppose the length of ladder is x m.

From the figure, we have

$\frac{18}{x} = \sin\ 30^\circ \quad \ldots\left[\because \sin = \frac{\text{Perpendicular}}{\text{Hypotenuse}}\right]$

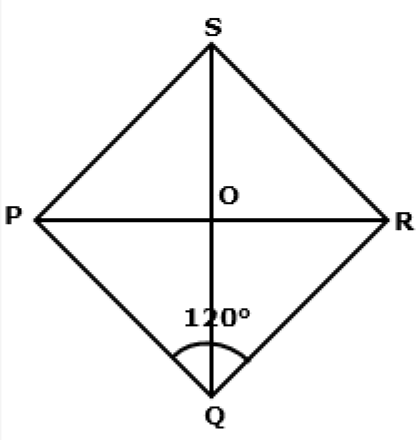
$\Rightarrow \frac{18}{x} = \frac{1}{2}$

$\Rightarrow x = 36$

Thus, the length of ladder is 36 m.

Solution 19.

Consider the following figure,



Perimeter of rhombus = 100 cm

$\Rightarrow PQ = QR = RS = SP = \frac{100}{4} = 25$ cm

Diagonals of a rhombus bisect each other ar right angles.

$\Rightarrow PO = OR$ and $QO = OS$

And, $\angle POQ = \angle ROQ = \angle ROS = \angle POS = 90°$

Also, diagonals bisect the angle at vertex.

$\Rightarrow \angle PQO = \frac{1}{2}\angle PQR = \frac{1}{2} \times 120° = 60°$

Now, in right ΔPOR,

$\sin(\angle PQO) = \frac{OP}{PQ}$

$\Rightarrow \sin 60° = \frac{OP}{25}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{OP}{25}$

$\Rightarrow OP = \frac{25\sqrt{3}}{2}$

$\therefore PR = 2 \times OP = 2 \times \frac{25\sqrt{3}}{2} = 25\sqrt{3}$ cm

Also, $\cos(\angle PQO) = \frac{OQ}{PQ}$

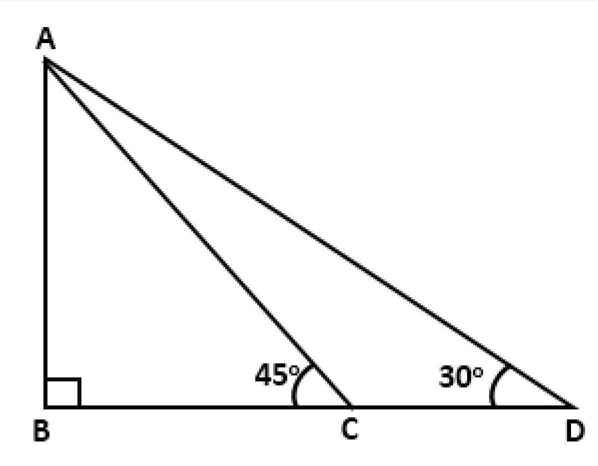
$\Rightarrow \cos 60° = \frac{OQ}{25}$

$\Rightarrow \frac{1}{2} = \frac{OQ}{25}$

$\Rightarrow OQ = \frac{25}{2}$

$\therefore SQ = 2 \times OQ = 2 \times \frac{25}{2} = 25$ cm

Solution 20.



In right ΔABC,

$\tan 45^\circ = \frac{AB}{BC}$

$\Rightarrow 1 = \frac{24}{BC}$

$\Rightarrow BC = 24$ m

In right ΔABD,

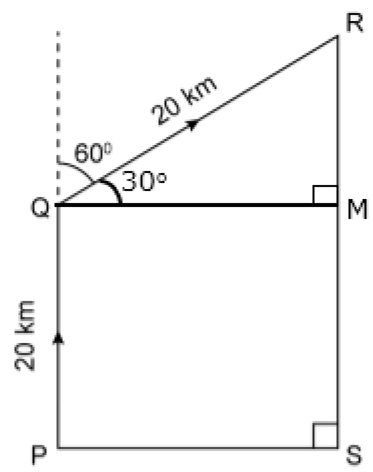
$\tan 30^\circ = \frac{AB}{BD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{24}{BD}$

$\Rightarrow BD = 24\sqrt{3}$ m

Now, $CD = BD - BC = 24\sqrt{3} - 24 = 24\left(\sqrt{3} - 1\right)$ m

Solution 21.



a. In right ΔRMQ,

$$\sin 30^\circ = \frac{RM}{RQ}$$

$$\Rightarrow \frac{1}{2} = \frac{RM}{20}$$

$$\Rightarrow RM = 10 \text{ km}$$

$\therefore$ The height of the rocket when it is at point R

$= RS$

$= RM + MS$

$= 10 \text{ km} + 20 \text{ km}$

$= 30 \text{ km}$

b. In right ΔRMQ,

$$\cos 30^\circ = \frac{QM}{RQ}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{QM}{20}$$

$$\Rightarrow QM = 10\sqrt{3} \text{ km}$$

$\therefore$ The horizontal distance of point S from P

$= PS$

$= QM$

$= 10\sqrt{3} \text{ km}$

Ex No: 27.3

Solution 1a.

$$\frac{\sin 62^\circ}{\cos 28^\circ} = \frac{\sin(90^\circ - 28^\circ)}{\cos 28^\circ} = \frac{\cos 28^\circ}{\cos 28^\circ} = 1$$

Solution 1b.

$$\frac{\sec 34^\circ}{\operatorname{cosec} 56^\circ} = \frac{\sec(90^\circ - 56^\circ)}{\operatorname{cosec} 56^\circ} = \frac{\operatorname{cosec} 56^\circ}{\operatorname{cosec} 56^\circ} = 1$$

Solution 1c.

$$\frac{\tan 12^\circ}{\cot 78^\circ} = \frac{\tan(90^\circ - 78^\circ)}{\cot 78^\circ} = \frac{\cot 78^\circ}{\cot 78^\circ} = 1$$

Solution 1d.

$$\frac{\sin 25^\circ \cos 43^\circ}{\sin 47^\circ \cos 65^\circ}$$

$$= \frac{\sin(90^\circ - 65^\circ)\cos(90^\circ - 47^\circ)}{\sin 47^\circ \cos 65^\circ}$$

$$= \frac{\cos 65^\circ \sin 47^\circ}{\sin 47^\circ \cos 65^\circ}$$

$$= 1$$

Solution 1e.

$$\frac{\sec 32^\circ \cot 26^\circ}{\tan 64^\circ \operatorname{cosec} 58^\circ}$$

$$= \frac{\sec(90^\circ - 58^\circ)\cot(90^\circ - 64^\circ)}{\tan 64^\circ \operatorname{cosec} 58^\circ}$$

$$= \frac{\operatorname{cosec} 58^\circ \cot 64^\circ}{\tan 64^\circ \operatorname{cosec} 58^\circ}$$

$$= 1$$

Solution 1f.

$$\frac{\cos 34^\circ \cos 33^\circ}{\sin 57^\circ \sin 56^\circ}$$
$$= \frac{\cos(90^\circ - 56^\circ)\cos(90^\circ - 57^\circ)}{\sin 57^\circ \sin 56^\circ}$$
$$= \frac{\sin 56^\circ \sin 57^\circ}{\sin 57^\circ \sin 56^\circ}$$
$$= 1$$

Solution 2a.

$$\sin 31^\circ - \cos 59^\circ$$
$$= \sin(90^\circ - 59^\circ) - \cos 59^\circ$$
$$= \cos 59^\circ - \cos 59^\circ$$
$$= 0$$

Solution 2b.

$$\cot 27^\circ - \tan 63^\circ$$
$$= \cot(90^\circ - 63^\circ) - \tan 63^\circ$$
$$= \tan 63^\circ - \tan 63^\circ$$
$$= 0$$

Solution 2c.

$$\text{cosec } 54^\circ - \sec 36^\circ$$
$$= \text{cosec}(90^\circ - 36^\circ) - \sec 36^\circ$$
$$= \sec 36^\circ - \sec 36^\circ$$
$$= 0$$

Solution 2d.

$$\sin 28^\circ \sec 62^\circ + \tan 49^\circ \tan 41^\circ$$
$$= \sin 28^\circ \sec(90^\circ - 28^\circ) + \tan 49^\circ \tan(90^\circ - 49^\circ)$$
$$= \sin 28^\circ \text{cosec } 28^\circ + \tan 49^\circ \cot 49^\circ$$
$$= \sin 28^\circ \times \frac{1}{\sin 28^\circ} + \tan 49^\circ \times \frac{1}{\tan 49^\circ}$$
$$= 1 + 1$$
$$= 2$$

Solution 2e.

$\sec 16° \tan 28° - \cot 62° \operatorname{cosec} 74°$

$= \sec(90° - 74°)\tan(90° - 62°) - \cot 62° \operatorname{cosec} 74°$

$= \operatorname{cosec} 74° \cot 62° - \cot 62° \operatorname{cosec} 74°$

$= 0$

Solution 2f.

$\sin 22° \cos 44° - \sin 46° \cos 68°$

$= \sin(90° - 68°)\cos(90° - 46°) - \sin 46° \cos 68°$

$= \cos 68° \sin 46° - \sin 46° \cos 68°$

$= 0$

Solution 3a.

$\frac{\sin 36°}{\cos 54°} + \frac{\sec 31°}{\operatorname{cosec} 59°}$

$= \frac{\sin(90° - 54°)}{\cos 54°} + \frac{\sec(90° - 59°)}{\operatorname{cosec} 59°}$

$= \frac{\cos 54°}{\cos 54°} + \frac{\operatorname{cosec} 59°}{\operatorname{cosec} 59°}$

$= 1 + 1$

$= 2$

Solution 3b.

$\frac{\tan 42°}{\cot 48°} - \frac{\cos 33°}{\sin 57°}$

$= \frac{\tan(90° - 48°)}{\cot 48°} - \frac{\cos(90° - 57°)}{\sin 57°}$

$= \frac{\cot 48°}{\cot 48°} - \frac{\sin 57°}{\sin 57°}$

$= 1 - 1$

$= 0$

Solution 3c.

$$\frac{2\sin 28^\circ}{\cos 62^\circ} + \frac{3\cot 49^\circ}{\tan 41^\circ}$$
$$= \frac{2\sin(90^\circ - 62^\circ)}{\cos 62^\circ} + \frac{3\cot(90^\circ - 41^\circ)}{\tan 41^\circ}$$
$$= \frac{2\cos 62^\circ}{\cos 62^\circ} + \frac{3\tan 41^\circ}{\tan 41^\circ}$$
$$= 2 + 3$$
$$= 5$$

Solution 3d.

$$\frac{5\sec 68^\circ}{\operatorname{cosec} 22^\circ} + \frac{3\sin 52^\circ \sec 38^\circ}{\cot 51^\circ \cot 39^\circ}$$
$$= \frac{5\sec(90^\circ - 22^\circ)}{\operatorname{cosec} 22^\circ} + \frac{3\sin 52^\circ \sec(90^\circ - 52^\circ)}{\cot 51^\circ \cot(90^\circ - 51^\circ)}$$
$$= \frac{5\operatorname{cosec} 22^\circ}{\operatorname{cosec} 22^\circ} + \frac{3\sin 52^\circ \operatorname{cosec} 52^\circ}{\cot 51^\circ \tan 51^\circ}$$
$$= 5 + \frac{3\sin 52^\circ \times \frac{1}{\sin 52^\circ}}{\cot 51^\circ \times \frac{1}{\cot 51^\circ}}$$
$$= 5 + \frac{3}{1}$$
$$= 5 + 3$$
$$= 8$$

Solution 4a.

$$\sin 65^\circ + \cot 59^\circ$$
$$= \sin(90^\circ - 25^\circ) + \cot(90^\circ - 31^\circ)$$
$$= \cos 25^\circ + \tan 31^\circ$$

Solution 4b.

$$\cos 72^\circ - \cos 88^\circ$$
$$= \cos(90^\circ - 18^\circ) - \cos(90^\circ - 2^\circ)$$
$$= \sin 18^\circ - \sin 2^\circ$$

Solution 4c.

$\text{cosec } 64° + \sec 70°$
$= \text{cosec } (90° - 26°) + \sec (90° - 20°)$
$= \sec 26° + \text{cosec } 20°$

Solution 4d.

$\tan 77° - \cot 63° + \sin 57°$
$= \tan (90° - 13°) - \cot (90° - 27°) + \sin (90° - 33°)$
$= \cot 13° - \tan 27° + \cos 33°$

Solution 4e.

$\sin 53° + \sec 66° - \sin 50°$
$= \sin (90° - 37°) + \sec (90° - 24°) - \sin (90° - 40°)$
$= \cos 37° + \text{cosec } 24° - \cos 40°$

Solution 4f.

$\cos 84° + \text{cosec } 69° - \cot 68°$
$= \cos (90° - 6°) + \text{cosec } (90° - 21°) - \cot (90° - 22°)$
$= \sin 6° + \sec 21° - \tan 22°$

Solution 5a.

$\sin 35° \sin 45° \sec 55° \sec 45°$
$= \sin (90° - 55°) \times \frac{1}{\sqrt{2}} \times \frac{1}{\cos 55°} \times \sqrt{2}$
$= \cos 55° \times \frac{1}{\cos 55°} \times \frac{1}{\sqrt{2}} \times \sqrt{2}$
$= 1$

Solution 5b.

$\cot 20° \cot 40° \cot 45° \cot 50° \cot 70°$
$= \cot (90° - 70°) \times \cot (90° - 50°) \times 1 \times \cot 50° \times \cot 70°$
$= \tan 70° \times \tan 50° \times \cot 50° \times \cot 70°$
$= \tan 70° \times \cot 70° \times \tan 50° \times \cot 50°$
$= \tan 70° \times \frac{1}{\tan 70°} \times \tan 50° \times \frac{1}{\tan 50°}$
$= 1$

Solution 5c.

$\cos 39° \cos 48° \cos 60° \operatorname{cosec} 42° \operatorname{cosec} 51°$

$= \cos(90° - 51°) \times \cos(90° - 42°) \times \frac{1}{2} \times \frac{1}{\sin 42°} \times \frac{1}{\sin 51°}$

$= \sin 51° \times \sin 42° \times \frac{1}{2} \times \frac{1}{\sin 42°} \times \frac{1}{\sin 51°}$

$= \frac{1}{2}$

Solution 5d.

$\sin(35° + \theta) - \cos(55° - \theta) - \tan(42° + \theta) + \cot(48° - \theta)$

$= \sin[90° - (55° - \theta)] - \cos(55° - \theta) - \tan[90° - (48° - \theta)] + \cot(48° - \theta)$

$= \cos(55° - \theta) - \cos(55° - \theta) - \cot(48° - \theta) + \cot(48° - \theta)$

$= 0$

Solution 5e.

$\tan(78° + \theta) + \operatorname{cosec}(42° + \theta) - \cot(12° - \theta) - \sec(48° - \theta)$

$= \tan[90° - (12° - \theta)] + \operatorname{cosec}[90° - (48° - \theta)] - \cot(12° - \theta) - \sec(48° - \theta)$

$= \cot(12° - \theta) + \sec(48° - \theta)] - \cot(12° - \theta) - \sec(48° - \theta)$

$= 0$

Solution 5f.

$\frac{3\sin 37°}{\cos 53°} - \frac{5 \operatorname{cosec} 39°}{\sec 51°} + \frac{4 \tan 23° \tan 37° \tan 67° \tan 53°}{\cos 17° \cos 67° \operatorname{cosec} 73° \operatorname{cosec} 23°}$

$= \frac{3 \sin(90° - 53°)}{\cos 53°} - \frac{5 \operatorname{cosec}(90° - 51°)}{\sec 51°}$

$+ \frac{4 \tan(90° - 67°) \tan(90° - 53°) \times \frac{1}{\cot 67°} \times \frac{1}{\cot 53°}}{\cos(90° - 73°) \cos(90° - 23°) \times \frac{1}{\sin 73°} \times \frac{1}{\sin 23°}}$

$= \frac{3 \cos 53°}{\cos 53°} - \frac{5 \sec 51°}{\sec 51°} + \frac{4 \cot 67° \cot 53° \times \frac{1}{\cot 67°} \times \frac{1}{\cot 53°}}{\sin 73° \sin 23° \times \frac{1}{\sin 73°} \times \frac{1}{\sin 23°}}$

$= 3 - 5 + 4$

$= 2$

Solution 5g.

$$\frac{\sin 0^\circ \sin 35^\circ \sin 55^\circ \sin 75^\circ}{\cos 22^\circ \cos 64^\circ \cos 68^\circ \cos 90^\circ}$$
$$= \frac{0 \times \sin 35^\circ \sin 55^\circ \sin 75^\circ}{\cos 22^\circ \cos 64^\circ \cos 68^\circ \times 0}$$
$$= 0$$

Solution 5h.

$$\frac{2 \sin 25^\circ \sin 35^\circ \sec 55^\circ \sec 65^\circ}{5 \tan 29^\circ \tan 45^\circ \tan 61^\circ} + \frac{3 \cos 20^\circ \cos 50^\circ \cot 70^\circ \cot 40^\circ}{5 \tan 20^\circ \tan 50^\circ \sin 70^\circ \sin 40^\circ}$$
$$= \frac{2 \sin (90^\circ - 65^\circ) \sin (90^\circ - 55^\circ) \sec 55^\circ \sec 65^\circ}{5 \tan (90^\circ - 61^\circ) \times 1 \times \tan 61^\circ}$$
$$+ \frac{3 \cos (90^\circ - 70^\circ) \cos (90^\circ - 40^\circ) \cot (90^\circ - 20^\circ) \cot (90^\circ - 50^\circ)}{5 \tan 20^\circ \tan 50^\circ \sin 70^\circ \sin 40^\circ}$$
$$= \frac{2 \cos 65^\circ \cos 55^\circ \times \frac{1}{\cos 55^\circ} \times \frac{1}{\sec 65^\circ}}{5 \cot 61^\circ \times 1 \times \frac{1}{\cot 61^\circ}} + \frac{3 \sin 70^\circ \sin 40^\circ \tan 20^\circ \tan 50^\circ}{5 \tan 20^\circ \tan 50^\circ \sin 70^\circ \sin 40^\circ}$$
$$= \frac{2}{5} + \frac{3}{5}$$
$$= \frac{5}{5}$$
$$= 1$$

Solution 5i.

$$\frac{3 \sin^2 40^\circ}{4 \cos^2 50^\circ} - \frac{\operatorname{cosec}^2 28^\circ}{4 \sec^2 62^\circ} + \frac{\cos 10^\circ \cos 25^\circ \cos 45^\circ \operatorname{cosec} 80^\circ}{2 \sin 15^\circ \sin 25^\circ \sin 45^\circ \sin 65^\circ \sec 75^\circ}$$
$$= \frac{3 \sin^2 (90^\circ - 50^\circ)}{4 \cos^2 50^\circ} - \frac{\operatorname{cosec}^2 (90^\circ - 62^\circ)}{4 \sec^2 62^\circ}$$
$$+ \frac{\cos (90^\circ - 80^\circ) \cos 25^\circ \times \frac{1}{\sqrt{2}} \times \frac{1}{\sin 80^\circ}}{2 \sin (90^\circ - 75^\circ) \times \frac{1}{\sqrt{2}} \times \sin (90^\circ - 25^\circ) \times \frac{1}{\cos 75^\circ}}$$
$$= \frac{3 \cos^2 50^\circ}{4 \cos^2 50^\circ} - \frac{\sec^2 62^\circ}{4 \sec^2 62^\circ} + \frac{\sin 80^\circ \times \cos 25^\circ \times \frac{1}{\sin 80^\circ}}{2 \cos 75^\circ \times \cos 25^\circ \times \frac{1}{\cos 75^\circ}}$$
$$= \frac{3}{4} - \frac{1}{4} + \frac{1}{2}$$
$$= \frac{1}{2} + \frac{1}{2}$$
$$= 1$$

Solution 5j.

$$\frac{5\cot 5^\circ \cot 15^\circ \cot 25^\circ \cot 35^\circ \cot 45^\circ}{7\tan 45^\circ \tan 55^\circ \tan 65^\circ \tan 75^\circ \tan 85} + \frac{2\operatorname{cosec} 12^\circ \operatorname{cosec} 24^\circ \cos 78^\circ \cos 66^\circ}{7\sin 14^\circ \sin 23^\circ \sec 76^\circ \sec 67^\circ}$$

$$= \frac{5\cot(90^\circ - 85^\circ)\cot(90^\circ - 75^\circ)\cot(90^\circ - 65^\circ)\cot(90^\circ - 55^\circ) \times 1}{7 \times 1 \times \tan 55^\circ \tan 65^\circ \tan 75^\circ \tan 85} + \frac{2\operatorname{cosec}(90^\circ - 78^\circ)\operatorname{cosec}(90^\circ - 66^\circ)\cos 78^\circ \cos 66^\circ}{7\sin(90^\circ - 76^\circ)\sin(90^\circ - 67^\circ)\sec 76^\circ \sec 67^\circ}$$

$$= \frac{5\tan 85^\circ \tan 75^\circ \tan 65^\circ \tan 55^\circ}{7 \times \tan 55^\circ \tan 65^\circ \tan 75^\circ \tan 85} + \frac{2\sec 78^\circ \sec 66^\circ \times \frac{1}{\sec 78^\circ} \times \frac{1}{\sec 66^\circ}}{7\cos 76^\circ \cos 67^\circ \times \frac{1}{\cos 76^\circ} \times \frac{1}{\cos 67^\circ}}$$

$$= \frac{5}{7} + \frac{2}{7}$$

$$= \frac{7}{7}$$

$$= 1$$

Solution 6.

$\cos 3\theta = \sin(\theta - 34^\circ)$

$\Rightarrow \sin(90^\circ - 3\theta) = \sin(\theta - 34^\circ)$

$\Rightarrow 90^\circ - 3\theta = \theta - 34^\circ$

$\Rightarrow 4\theta = 124^\circ$

$\Rightarrow \theta = 31^\circ$

Solution 7.

$\tan 4\theta = \cot(\theta + 20^\circ)$

$\Rightarrow \cot(90^\circ - 4\theta) = \cot(\theta + 20^\circ)$

$\Rightarrow 90^\circ - 4\theta = \theta + 20^\circ$

$\Rightarrow 5\theta = 70^\circ$

$\Rightarrow \theta = 14^\circ$

Solution 8.

$\sec 2\theta = \operatorname{cosec} 3\theta$

$\Rightarrow \sec 2\theta = \sec(90^\circ - 3\theta)$

$\Rightarrow 2\theta = 90^\circ - 3\theta$

$\Rightarrow 5\theta = 90^\circ$

$\Rightarrow \theta = 18^\circ$

Solution 9.

$\sin(\theta - 15^\circ) = \cos(\theta - 25^\circ)$

$\Rightarrow \cos[90^\circ - (\theta - 15^\circ)] = \cos(\theta - 25^\circ)$

$\Rightarrow 90^\circ - (\theta - 15^\circ) = \theta - 25^\circ$

$\Rightarrow 105^\circ - \theta = \theta - 25^\circ$

$\Rightarrow 2\theta = 130^\circ$

$\Rightarrow \theta = 65^\circ$

Solution 10.

Since A, B and C are interior angles of ΔABC,

$A + B + C = 180^\circ$

$\Rightarrow A + B = 180^\circ - C$

Now,

$$\text{L.H.S.} = \sin\left(\frac{A+B}{2}\right)$$

$$= \sin\left(\frac{180^\circ - C}{2}\right)$$

$$= \sin\left(90^\circ - \frac{C}{2}\right)$$

$$= \cos\frac{C}{2}$$

$= \text{R.H.S.}$

Solution 11.

since P, Q and R are the interior angles of ΔPQR,

$P + Q + R = 180^\circ$

$\Rightarrow Q + R = 180^\circ - P$

Now,

$$\text{L.H.S.} = \cot\left(\frac{Q+R}{2}\right)$$

$$= \cot\left(\frac{180^\circ - P}{2}\right)$$

$$= \cot\left(90^\circ - \frac{P}{2}\right)$$

$$= \tan\frac{P}{2}$$

$= \text{R.H.S.}$

Solution 12.

$\cos\theta = \sin 60^\circ$

$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow \cos\theta = \cos 30^\circ$

$\Rightarrow \theta = 30^\circ$

Now,

$1 - 2\sin^2\theta = 1 - 2\sin^2 30^\circ = 1 - 2 \times \left(\frac{1}{2}\right)^2 = 1 - 2 \times \frac{1}{4} = 1 - \frac{1}{2} = \frac{1}{2}$

Solution 13.

$\sec\theta = \operatorname{cosec} 30^\circ$

$\Rightarrow \sec\theta = 2$

$\Rightarrow \sec\theta = \sec 60^\circ$

$\Rightarrow \theta = 60^\circ$

Now,

$4\sin^2\theta - 2\cos^2\theta$

$= 4\sin^2 60^\circ - 2\cos^2 60^\circ$

$= 4 \times \left(\frac{\sqrt{3}}{2}\right)^2 - 2 \times \left(\frac{1}{2}\right)^2$

$= 4 \times \frac{3}{4} - 2 \times \frac{1}{4}$

$= 3 - \frac{1}{2}$

$= \frac{6-1}{2}$

$= \frac{5}{2}$

Solution 14a.

$\text{L.H.S.} = \tan\theta \tan(90^\circ - \theta)$

$= \cot(90^\circ - \theta) \times \cot\theta$

$= \cot\theta \cot(90^\circ - \theta)$

$= \text{R.H.S.}$

Solution 14b.

$$\text{L.H.S.} = \sin 58^\circ \sec 32^\circ + \cos 58^\circ \operatorname{cosec} 32^\circ$$

$$= \sin(90^\circ - 32^\circ) \times \frac{1}{\cos 32^\circ} + \cos(90^\circ - 32^\circ) \times \frac{1}{\sin 32^\circ}$$

$$= \cos 32^\circ \times \frac{1}{\cos 32^\circ} + \sin 32^\circ \times \frac{1}{\sin 32^\circ}$$

$$= 1 + 1$$

$$= 2$$

$$= \text{R.H.S.}$$

Solution 14c.

$$\text{L.H.S.} = \frac{\tan(90^\circ - \theta)\cot\theta}{\operatorname{cosec}^2\theta}$$

$$= \frac{\cot\theta \times \cot\theta}{\operatorname{cosec}^2\theta}$$

$$= \frac{\cot^2\theta}{\operatorname{cosec}^2\theta}$$

$$= \frac{\frac{\cos^2\theta}{\sin^2\theta}}{\frac{1}{\sin^2\theta}}$$

$$= \cos^2\theta$$

$$= \text{R.H.S.}$$

Solution 14d.

$$\text{L.H.S.} = \sin^2 30^\circ + \cos^2 30^\circ$$

$$= \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= \frac{1}{4} + \frac{3}{4}$$

$$= \frac{4}{4}$$

$$= 1$$

$$= \frac{1}{2} \times \sec 60^\circ$$

$$= \text{R.H.S.}$$

Solution 15.

$A + B = 90^\circ$

$\Rightarrow B = 90^\circ - A$

$$\text{L.H.S.} = \frac{\tan A \tan B + \tan A \cot B}{\sin A \sec B} - \frac{\sin^2 B}{\cos^2 A}$$

$$= \frac{\tan A \tan(90^\circ - A) + \tan A \cot(90^\circ - A)}{\sin A \sec(90^\circ - A)} - \frac{\sin^2(90^\circ - A)}{\cos^2 A}$$

$$= \frac{\tan A \cot A + \tan A \tan A}{\sin A \operatorname{cosec} A} - \frac{\cos^2 A}{\cos^2 A}$$

$$= \frac{1 + \tan^2 A}{1} - 1$$

$$= 1 + \tan^2 A - 1$$

$$= \tan^2 A$$

$$= \text{R.H.S.}$$